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DISPERSION OF LAMB WAVES IN MULTILAYER STRUCTURES

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Abstract: Low cost, the possibility of online monitoring and high sensitivity distinguish the method of structural monitoring using Lamb waves from other available methods. Structural analysis based on Lamb waves in heterogeneous materials requires fundamental knowledge of the behavior of Lamb waves in such materials. This basic knowledge is critical for signal processing in determining possible damage that can be detected by the propagating wave. Recently, Lamb wave methods have been used to simultaneously survey large areas of composite structures. However, such methods are more complex than traditional ultrasonic testing because Lamb waves have dispersive characteristics, namely, the wave speed varies depending on the frequency, modes and thickness of the plates. Experimentally measured group velocities of Lamb waves in composite materials with anisotropic characteristics do not coincide with theoretical group velocities, which are calculated using the dispersion equation of Lamb waves. This discrepancy arises because in anisotropic materials there is an angle between the direction of the group velocity and the direction of the phase velocity. This work investigates the propagation characteristics of Lamb waves in composites, focusing on group velocity and characteristic wave curves. For symmetric laminates, a robust method is proposed by imposing boundary conditions on the mid-plane and top surface to separate symmetric and antisymmetric wave modes. The dispersive and anisotropic behavior of Lamb waves in two different types of symmetrical laminates is theoretically studied in detail. The dispersion of Lamb waves was studied for 10 symmetric and asymmetric modes. It is shown that only fundamental modes are not characterized by a cutoff frequency, which indicates the interaction of fundamental modes with composite layers in the low-frequency range. A high level of group velocity dispersion was discovered for the SH0 and S0 modes. It is concluded that in isotropic laminates, dispersion is characteristic of symmetric modes. It is shown that the frequency dependence of the group velocity of Lamb waves of laminar composites can be represented in polynomial form.

Keywords: Lamb wave, group velocity dispersion, wavelet analysis, laminar composites.

ДИСПЕРСІЯ ХВИЛЬ ЛЕМБА У БАГАТОШАРОВИХ СТРУКТУРАХ

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Анотація: З різних доступних підходів структурного моніторингу композитних матеріалів використання хвиль Лемба є дуже цікавим методом через його низьку вартість, онлайн-моніторинг і високу чутливість. Структурний аналіз на основі хвиль Лемба у гетерогенних матеріалах потребує фундаментальних знань про поведінку хвиль у таких матеріалах. Ці базові знання мають вирішальне значення для обробки сигналів при визначенні можливих пошкоджень, які можуть бути виявлені хвилею, що поширюється. Останнім часом методи хвиль Лемба стали використовуватися для одночасного обстеження великої площі композитних конструкцій. Однак такі методи складніші за традиційні ультразвукові випробування, оскільки хвилі мають дисперсійні характеристики, тобто. Швидкість хвилі змінюється в залежності від частоти, мод та товщини пластин. Експериментально виміряні групові швидкості хвиль в композиційних матеріалах з анізотропними характеристиками не збігаються з груповими теоретичними швидкостями, які розраховуються за допомогою дисперсійного рівняння хвиль Лемба. Ця невідповідність виникає через те, що в анізотропних матеріалах існує кут між напрямком групової швидкості та напрямком фазової швидкості. У цій роботі досліджено характеристики поширення хвиль Лемба в композитах з упором на групову швидкість і характерні хвилеві криві. Для симетричних ламінатів пропонується надійний метод шляхом накладання граничних умов на середню площину та верхню поверхню для поділу симетричних та антисиметричних хвильових мод. Детально теоретично досліджено дисперсійну та анізотропну поведінку хвиль Лемба у двох різних типах симетричних ламінатів. Дисперсія хвиль при поширенні між шарами в ламінарних композитах досліджена для 10 симетричних та асиметричних мод. Показано, що лише основні моди не характеризуються частотою зрізу, що вказує на взаємодію фундаментальних мод із шарами композиту у низькочастотному діапазоні. Виявлено високий рівень дисперсії групової швидкості мод SH_0 і S_0. Зроблено висновок, що в ізотропних ламінатах дисперсія при поширенні хвильового процесу характерна для симетричних мод. Показано, що частотна залежність групової швидкості хвиль Лемба ламінарних композитів може бути представлена в поліноміальній формі.

Ключові слова: хвиля Лемба, дисперсія групової швидкості, вейвлет-аналіз, ламінарні композити.

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1 INTRODUCTION

Non-destructive testing [1, 2] and structural health monitoring [3, 4] have traditionally been the two main wavelet transform methods for assessing the integrity and degradation of composite systems widely used in construction. Implementation of an active diagnostic procedure that uses ultrasonic waves to detect damage, localize and subsequently evaluate damage involves understanding the propagation characteristics of these waves in composites.

Factors that influence the speed of wave mode propagation include the laminate laying features, wave direction, frequency and interface conditions. The dependence of the wave front speed on frequency leads to the need for a detailed study of the dispersion properties of directed waves propagating along the plane of an elastic composite plate with boundaries free from mechanical stress (Lamb waves).

As a rule, the direction of waves in laminar composites is classified by polarization perpendicular to the composite plate (symmetric S waves, antisymmetric A waves) and parallel to the plate (shear horizontal SH waves).

For waves propagating in multilayer composites, wave interactions depend on the properties of the constituents, geometry, direction of propagation, frequency, and interfacial conditions. If the wavelengths significantly exceed the dimensions of the constituent composites (the diameters of the fibers and the distance between them), each plate can be considered as an equivalent homogeneous orthotropic or transversally isotropic material with an axis of symmetry parallel to the fibers.

2 ANALYSIS OF PUBLICATIONS

The study of Lamb waves (wavelet analysis) in composites [5] is most often carried out using two theoretical approaches, namely, exact solutions using three-dimensional elasticity theory and approximate solutions using plate theory.

Saito and Okabe [6] investigated the dispersion relation of Lamb waves propagating in a cross-ply CFRP laminate. Using a formalism of the multi-layer Lamb wave model, they compared a homogeneous single-layer model and multi-layer models.

Liu and Huang [7] examined the effect of inclusion shapes, inclusion contents, inclusion elastic constants, and plate thickness on the dispersion relations and modes of wave propagation in inclusion-reinforced composite plates. They determined the dispersion relations and the modal patterns of Lamb waves using the dynamic stiffness matrix method.

Orta et al. [8] introduced the new computational framework which allows to estimate the dispersion curves for the first nine symmetric and nine anti-symmetric Lamb modes. Analytically calculated dispersion curves using 5-SDT for different propagation directions and polar plots for selected frequency of different materials are compared with the results from both the semi analytical finite element method, and lower order shear deformation theories.

Ma et al. [9] constructed dispersion relations using the formulas of reverberation rays in a three-dimensional Cartesian coordinate, and numerically solved the transcendental equations using an improved mode tracking method.

Peddeti and Santhanam [10] formulated a semi-analytical finite element method (for the acoustoelastic problem of guided waves in weakly nonlinear elastic plates). It was shown that the formulation of this method provides phase velocity dispersion curve results identical to the results obtained for the problem of a plate under uniaxial and uniform tensile stress.

The character of the elastic waves causes that damage detection based on the analysis of the dynamic response of an interrogated structure becomes rather difficult [11, 12].

However, in a relatively small number of studies, dispersions of not phase, but group velocities of Lamb waves are considered [13-17].

So, the knowledge of moduli and group velocity dispersion enables the optimal location of the sensors in order to detect the potential damage.

The purpose of this work is to study the group velocity dispersion of symmetric and antisymmetric Lamb waves in laminar composites with different stacking structures.

3 PURPOSE AND OBJECTIVES OF THE STUDY

In general, transition waves propagating in anisotropic composites cause disturbances for all three displacement components. It is necessary to separately analyze the propagation of waves along the symmetry axes, namely, to take into account the splitting of *S*-, *A*- and *SH*-waves. The ultimate goal of the study is to compare the polynomial and exponential forms of the dispersion law for laminar composites. A Cartesian coordinate system is used in which the *z*-axis is perpendicular to the mid-plane of the composite laminate. The distance between the two outer surfaces of the laminate is $z = \pm d/2$. Let us consider the case of propagation of a packet of Lamb waves in the direction of δ. Each layer of the composite laminate is considered as a monoclinic material with a plane of symmetry $(x-y)$. The relationship between mechanical stress and deformation takes the following matrix form

$$A_i = G_{ki} D_k , \tag{1}$$

where $\{(a_i=\sigma_i)\cap(d_k=\varepsilon_k)|i,k\in(x,y,z)\}\cap\{(a_i=\tau_i)\cap(d_k=\gamma_k)|i,k\in(yz,xz,xy)\}$ are the matrices *A* and *D* coefficients; *G* is the stiffness matrix.

The equations of motion in the absence of body forces are governed by

$$\sigma_{x,x} + \tau_{xy,y} + \tau_{xz,z} = \rho \ddot{\alpha}_1 \tag{2}$$

$$\tau_{xy,x} + \sigma_{y,y} + \tau_{yz,z} = \rho \ddot{\alpha}_2 \tag{3}$$

$$\tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} = \rho \ddot{\alpha}_3 \tag{4}$$

where ρ is the mass density of the lamina, and dot denotes time derivative; α_1, α_2 and α_3 are the displacements in the x, y and $z-$ directions.

Boundary conditions on the top and bottom surfaces of the laminate

$$\sigma_z = \tau_{x,z} = \tau_{y,z} = 0, at\ z = \pm\delta/2 \tag{5}$$

Lamb waves can be considered as standing waves in the *z*-direction of the plate. The result of this assumption is a model of wave motion in the form of a superposition of plane harmonic waves. Each plane harmonic wave moving in the k direction is represented by displacement coefficients

$$\{\alpha_1,\alpha_2,\alpha_3\} = \{\beta_1(z),\beta_2(z),\beta_3(z)\} exp\{i(k_x x + k_y y) - \omega t\}, \tag{6}$$

where $k = [k_x, k_y]^T$ and its magnitude $k = |k| = \omega/\upsilon_p = 2\pi/\lambda$ is the wave number; ω is the angular frequency; λ is the wavelength and υ_p is the phase velocity. In the *x*-*y* plane, $k = k[\cos\eta, \sin\eta]^T$, where η is the direction of wave propagation.

In an off-axis laminar composite plate, solutions to the equation of motion can be simply separated into symmetric and antisymmetric waves. This consideration allows us to write down a fairly simple analytical representation

$$\begin{aligned}&\beta_{1,s}=E_s cosz,\beta_{2,s}=F_s cosz,\beta_{3,s}=G_s cosz\\&\beta_{1,a}=E_a sinz,\beta_{2,a}=F_a sinz,\beta_{3,a}=G_s sinz\end{aligned}, \tag{7}$$

where μ is the variable to be determined by Lamb wave kinematics; subscripts "s" and "a" represent symmetric and antisymmetric modes, respectively.

Substituting equation (7) into the equations of symmetrical wave motion, leads to an expression in matrix form

$$\begin{bmatrix}\Lambda_{11}-\rho\omega^2 & \Lambda_{12} & \Lambda_{13}\\ \Lambda_{12} & \Lambda_{22}-\rho\omega^2 & \Lambda_{23}\\ \bar{\Lambda}_{13} & \bar{\Lambda}_{23} & \Lambda_{33}-\rho\omega^2\end{bmatrix}\begin{Bmatrix}E_s\\F_s\\G_s\end{Bmatrix}=0, \tag{8}$$

where the overbar denotes complex conjugation.

The relationship between the elements of matrix $(\Lambda-\rho\omega^2 I)$ (8), stiffness matrix, and 3×3 identity matrix I have a polynomial form.

Nontrivial solutions E_s, F_s and G_s in equation (8) lead to the following sixth-order polynomial in μ

$$\mu^6+e_1\mu^4+e_2\mu^2+e_3=0, \tag{9}$$

where e_i $(i=1,2,3)$ are real-valued coefficients of G_{ij}, $\boldsymbol{k}$, and $\rho\omega^2$.

For each fixed μ_k (k = 1,2,3), solutions E_s, F_s and G_s are interrelated according to the equations

$$B_s=\frac{(\Lambda_{11}-\rho\omega^2)\Lambda_{23}-\Lambda_{12}\Lambda_{13}}{\Lambda_{13}(\Lambda_{22}-\rho\omega^2)-\Lambda_{12}\Lambda_{23}}E_s=RE_s \tag{10}$$

$$G_s=\frac{\Lambda_{12}^2-(\Lambda_{11}-\rho\omega^2)(\Lambda_{22}-\rho\omega^2)}{\Lambda_{13}(\Lambda_{22}-\rho\omega^2)-\Lambda_{12}\Lambda_{23}}E_s=iSE_s. \tag{11}$$

Antisymmetric modes make it possible to obtain similar relations: $F_a=RE_a$ and $G_a=-iSE_a$.

Equations (5), (7), (10), and (11) can be rearranged as

$$\left(\sigma_z,\tau_{yz},\sigma_{xz}\right)\Big|_{z=\delta/2}=\sum_{j=1}^{3}\Big[H_{1j}sin(\mu_j z+\varphi),H_{2j}cos(\mu_j z+\varphi),$$

$$H_{3j}cos(\mu_j z+\varphi)\Big]A_j=0, \tag{12}$$

where phases $\varphi=0$ and $\varphi=\pi/2$ correspond to symmetrical and asymmetrical Lamb wave modes, respectively, and

$$H_{1j}=G_{13}k_x+G_{23}k_yR_j+G_{33}\mu_jS_j+G_{36}(k_y+k_xR_j) \tag{13}$$

$$H_{2j}=G_{44}(\mu_jR_j+k_yS_j)+G_{45}(\mu_j+k_xS_j) \tag{14}$$

$$H_{3j}=G_{45}(\mu_jR_j+k_yS_j)+G_{55}(\mu_j+k_xS_j). \tag{15}$$

The simplified Lamb wave propagation model assumes ideal coupling between layers of the laminated composite in the z-direction. Accounting for laminate heterogeneity requires an exponential change in the displacement components

$$\beta_1 = Eexp\left(i\mu z\right), \beta_2 = Fexp\left(i\mu z\right), \beta_3 = -Gexp\left(i\mu z\right). \tag{16}$$

For each μ_i, values F and G can be expressed in terms of E as $F_i = R_i E_i$ and $G_i = -S_i E_i$ $(i=\overline{1,6})$. In addition, $R_{j+1} = R_j$ and $S_{j+1} = -S_j$.

Finally, the equation of motion for each layer is

$$\left\{\beta_1, \beta_2, \beta_3\right\} = exp\left\{i\left[\left(k_x x + k_y y\right) - \omega t\right]\right\} \sum_{j=1}^{6} E_j \left\{1, R_j, S_j\right\} exp\left(i\mu_i z\right). \tag{17}$$

The stress components σ_z, τ_{yz} and τ_{xz} between adjacent composite layers can be expressed as

$$\left\{\sigma_z, \tau_{yz}, \tau_{xz}\right\} = ikexp\left\{i\left[\left(k_x x + k_y y\right) - \omega t\right]\right\} \sum_{j=1}^{6} E_j \left\{H_{1j}, H_{2j}, H_{3j}\right\} exp\left(i\mu_i z\right). \tag{18}$$

By imposing displacement and stress continuity conditions along the interfaces between laminate layers. The solution of equation (18) leads to the dispersion relations of Lamb waves in symmetrical laminates.

Implicit functional forms J $(\omega, k) = 0$ and J $(\omega, k, \eta) = 0$ allow us to represent the dispersion relation between ω and k. These relations can be solved explicitly in the form of real roots of $\omega = \Omega(k)$, or $\omega = \Omega(k, \eta)$.

The phase velocity of plane waves is defined as

$$\upsilon_p = \left(\frac{\omega}{k}\right)\frac{k}{|k|} = \left(\frac{\omega}{k^2}\right)k. \tag{19}$$

The group velocity, determined from the envelopes of the wave packet, can be calculated using the implicit function G

$$\upsilon_g = -\frac{\partial J / \partial k}{\partial J / \partial \omega} \tag{20}$$

Cartesian projections of group velocity are determined by $grad_k\, \Omega$

$$\begin{Bmatrix} \upsilon_{gx} \\ \upsilon_{gy} \end{Bmatrix} = \begin{bmatrix} \cos\eta & -\sin\eta \\ \sin\eta & \cos\eta \end{bmatrix} \begin{Bmatrix} \partial\Omega / \partial k \\ \partial\Omega / k\partial\eta \end{Bmatrix}. \tag{21}$$

The current point on the wave-front curve represents the distance traveled by the elastic disturbance per unit time. Thus, the wave-front curve determines the location of the wave-front per unit time from the disturbance emitted by the point source at the initial time. Thus, waveforms are of great importance for detecting mechanical damage in laminar composites.

The relationship between the slowness curve and the direction of the group velocity allows wave curves to be calculated. The dispersion law of each Lamb wave mode can be expressed as an explicit function of $\Omega\,(k, \eta)$. The slowness curve is geometrically a level surface of $\Omega\,(k, \eta)$ at $\Omega_0(k, \eta) = \omega_0$. Differentiating both sides of the equation with respect to η yields

$$\frac{\partial\Omega}{\partial k}\frac{\partial k}{\partial\eta} + \frac{\partial\Omega}{\partial\eta} = 0. \tag{22}$$

The group velocity dispersion for a given direction of wave propagation η_1 can be obtained by using two dispersion relations $\omega = \Omega\,(k,\eta)$, the directions of which differ slightly $\eta_1 \pm \Delta\eta\,/2$. Then derivative term $\partial\Omega\,/\partial\eta$ can be approximated by finite central difference

$$\left.\frac{\partial\Omega}{\partial\eta}\right|_{\eta=\eta_1} \cong \frac{\Omega(k)\big|_{\eta_1+\Delta\eta_{/2}} - \Omega(k)\big|_{\eta_1-\Delta\eta_{/2}}}{\Delta\eta}. \tag{23}$$

The dispersion contribution $\partial\Omega\,/\partial k$ can be calculated using the calculation formula

$$\left.\frac{\partial\Omega}{\partial k}\right|_{\omega=\omega_1} \cong \frac{\omega_2-\omega_1}{k_2(\eta)-k_1(\eta)}.$$

4 RESEARCH RESULTS

The laminated composite material used in this study was characterized by the following properties: tensile stiffness E_1 = 1.276·10^{11} Pa, $E_2 = E_3$ = 1.13·10^{10} Pa; shear moduli G_{12} = 5.97·10^9 Pa; G_{13} = 5.97·10^9 Pa; G_{23} = 5.97·10^9 Pa; Poisson's ratios ν_{12} = 0.3, ν_{13} = 0.3; ν_{23} = 0.34; density ρ = 1.578·10^3 kg/m^3; stacking sequences $[+45_6/-45_6]_s$ (specimen A_1), $[+45/-45/0/90]_s$ (specimen A_2).

Spectral dependences of the dimensionless group velocity $\upsilon'_g = \upsilon_g/\upsilon_T$ for fixed values of the dimensionless frequency $f' = \omega\delta/\upsilon_T$ along the θ direction of laminates A_1 and A_2 are given in Tables 1 - 4. The value υ_T defined as $(G_{12}/\rho)^{0.5}$ is the transverse wave velocity in lamina (associated with shear in plane).

Table 1

Spectral profile of Lamb waves for laminate A_1
(symmetric modes)

f'	υ'_g		f'	υ'_g		
	S_0	SH_0		S_1	S_2	SH_2
0.5	3.325	2.384	5.0	0.962	0.001	0.001
1.0	3.218	2.321	5.6	0.921	0.002	0.003
1.5	3.085	2.305	6.2	0.824	1.512	1.264
2.0	2.798	2.208	6.8	0.841	2.358	1.587
2.5	2.237	2.126	7.4	0.935	2.857	1.698
3.0	1.749	2.111	8.0	1.045	3.042	1.762
4.0	0.387	1.564	8.6	1.018	3.110	1.852
5.0	1.400	0.631	9.2	1.089	3.043	1.964
6.0	1.310	0.735	9.5	1.088	3.002	1.993
7.0	1.182	0.786	10.1	1.070	2.804	2.057
8.0	1.087	0.811	10.8	0.993	2.220	2.125
9.0	1.010	0.832	11.4	0.968	0.995	2.173
10.0	1.000	0.846	12.0	0.970	0.484	2.186

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Table 2

Spectral profile of Lamb waves for laminate A_1
(asymmetrical modes)

f'	υ'_g	f'	υ'_g	f'	υ'_g		
	A_0		A_1		A_2	A_3	SH_3
0.5	0.651	2.5	1.882	8.5	0.593	0.003	0.001
1.0	0.847	2.9	2.456	8.6	0.612	0.227	0.001
1.5	0.851	3.3	2.614	8.7	0.715	0.418	0.002
2.0	0.856	3.7	2.913	8.8	0.783	0.623	0.003
2.5	0.623	4.1	3.111	8.9	0.805	0.701	0.003
3.0	0.678	4.5	3.152	9.0	0.890	0.862	0.004
3.5	0.699	4.9	3.112	9.1	0.904	0.871	0.125
4.0	0.734	5.3	3.087	9.2	0.928	0.885	0.364
4.5	0.790	5.7	2.924	9.3	0.957	0.889	0.541
5.0	0.802	6.1	2.631	9.4	0.981	0.896	0.683
5.5	0.813	6.5	2.185	9.5	1.061	0.900	0.754
6.0	0.845	6.9	1.598	9.6	1.082	0.882	0.974
6.5	0.887	7.3	1.273	9.7	1.106	0.874	1.116
7.0	0.902	7.7	0.832	9.8	1.125	0.856	1.277
7.5	0.883	8.1	0.401	9.9	1.143	0.830	1.452
8.0	0.879	8.5	0.368	10.0	1.162	0.795	1.5833
8.5	0.872	8.9	0.420	10.1	1.175	0.791	1.986
9.0	0.870	9.3	0.468	10.2	1.188	0.784	2.178
9.5	0.868	9.7	0.502	10.3	1.205	0.781	2.376
10.0	0.867	10.2	0.539	10.7	1.203	0.791	2.715

Table 3

Spectral profile of Lamb waves for laminate A_2
(symmetric modes)

f'	υ'_g		f'	υ'_g		
	S_0	SH_0		S_1	S_2	SH_2
0.5	3.042	1.803	5.0	1.527	0.041	0.005
1.0	3.005	1.752	5.6	1.832	1.184	0.679
1.5	2.910	1.685	6.2	1.709	0.913	1.563
2.0	2.805	1.599	6.8	1.564	1.286	2.037
2.5	2.609	1.485	7.4	1.301	1.701	2.311
3.0	2.308	1.361	8.0	1.105	1.723	2.325
4.0	0.726	1.100	8.6	0.984	1.600	2.297
5.0	0.948	0.556	9.2	0.826	1.417	2.137
6.0	0.911	0.674	9.5	0.794	1.284	2.000
7.0	0.926	0.725	10.1	0.731	1.142	1.806
8.0	0.937	0.792	10.8	0.701	0.984	1.658
9.0	0.945	0.805	11.4	0.702	0.900	1.052
10.0	0.954	0.815	12.0	0.704	0.898	0.854

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Table 4

Spectral profile of Lamb waves for laminate A_2
(asymmetrical modes)

f'	υ'_g	f'	υ'_g	f'	υ'_g		
	A_0		A_1		A_2	A_3	SH_3
0.5	0.898	2.5	1.218	8.5	0.924	0.003	0.002
1.0	0.898	2.9	1.530	8.6	0.895	0.008	0.164
1.5	0.897	3.3	1.809	8.7	0.861	0.012	0.308
2.0	0.897	3.7	2.184	8.8	0.837	0.016	0.407
2.5	0.897	4.1	2.394	8.9	0.820	0.021	0.593
3.0	0.896	4.5	2.426	9.0	0.815	0.028	0.699
3.5	0.895	4.9	2.385	9.1	0.793	0.089	0.715
4.0	0.894	5.3	2.288	9.2	0.765	0.187	0.805
4.5	0.894	5.7	2.235	9.3	0.737	0.352	0.881
5.0	0.893	6.1	1.980	9.4	0.718	0.605	0.973
5.5	0.893	6.5	1.684	9.5	0.694	0.831	1.113
6.0	0.893	6.9	1.295	9.6	0.711	0.927	1.188
6.5	0.892	7.3	1.064	9.7	0.725	1.164	1.246
7.0	0.892	7.7	0.845	9.8	0.740	1.235	1.358
7.5	0.891	8.1	0.555	9.9	0.756	1.380	1.455
8.0	0.891	8.5	0.485	10.0	0.768	1.486	1.557
8.5	0.891	8.9	0.316	10.1	0.773	1.604	1.618
9.0	0.890	9.3	0.484	10.2	0.791	1.728	1.735
9.5	0.890	9.7	0.587	10.3	0.804	1.872	1.882
10.0	0.890	10.2	0.615	10.7	0.809	1.914	1.912

5 DISCUSSION OF RESEARCH RESULTS

Five symmetric (Tables 1 and 3) and five asymmetric (Tables 2 and 4) modes illustrate the dispersion of Lamb waves curves in the layered composites A_1 and A_2. The analysis showed that only the fundamental modes (A_0, S_0 and SH_0) are not characterized by a cutoff frequency. This fact indicates the interactionof fundamental modes with composite layers in the low-frequency range. The frequency $\omega' = x\delta/\upsilon_T$ can be considered as the boundary frequency for SH_0 and S_0 modes, which have low dispersion in the range $\omega < \omega'$.

The calculation results show that the A_0 mode provides higher resolution than the S_0 and SH_0 modes. One of the reasons can be pointed to the fact that the A_0 mode wavelength is always shorter than that of the S_0 mode, especially in the low frequency range. Lamb wave propagation in a relatively thick symmetrical corner laminate $(+45_6/-45_6)_s$ for the high frequency range has a rather complex behaviour.

The SH_0 and S_0 modes are characterized by a fairly high level of group velocity dispersion. Analysis of the numerical data allows us to conclude that there is a higher level of symmetric mode dispersion for the quasi-isotropic laminate A_2 $(+45/-45/0/90)_s$. On the other hand, the dispersion of the antisymmetric wave mode A_0 in both laminates is weaker for
$\omega' \neq 1$.

The results of calculations of group velocity dispersion surfaces for wave modes in the laminar composites used make it possible to represent the polynomial dependence

$\upsilon'_g = \upsilon'_g(f')$ in matrix form

$$\upsilon'_{g,i} = d_{ik}\left(f'\right)^k, k = 1,\ldots,5, i \in \left(S_{0,A_1}, S_{0,A_2}, SH_{0,A_1}, SH_{0,A_2}\right)$$

$$d_{ik} = \begin{pmatrix} 0,076 & -0,631 & 1,467 & -1,514 & 3,798 \\ -0,009 & 0,182 & -1,096 & 1,813 & 2,264 \\ -0,004 & 0,092 & -0,0637 & 1,231 & 1,759 \\ -0,002 & 0,047 & -0,307 & 0,0469 & 1,584 \end{pmatrix}.$$

6 CONCLUSIONS

Exact solutions of Lamb waves in a plate can be established on the basis of three-dimensional elasticity theory and subsequently extended to a laminate with an arbitrary structure. For symmetrical laminates, a reliable wave mode separation method is used. A numerical method for obtaining group velocity dispersions and wave curves is proposed. The dispersions and characteristic wave curves of Lamb waves are analyzed for two types of laminates. The proposed methods effectively model the dispersive and anisotropic behavior of Lamb waves in laminates. It was found that the A_0 mode has the best characteristics for structural monitoring of laminar composites.

The speed of propagation of multi-frequency components within the wave packet remains almost unchanged, which causes only slight deformation of the wave packet shape when moving in the composite layers. In addition, the significantly low attenuation of A_0 mode and high sensitivity to the growth of delamination in the sample indicate the practical value of using symmetric modes as a diagnostic tool.

7 ETHICAL DECLARATIONS

The author has no relevant financial or non-financial interests to report.

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