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CALCULATION OF A CULVER PIPE BY THE NUMERICAL-ANALYTICAL BOUNDARY ELEMENTS METHOD

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Abstract: The methodology for applying the numerical-analytical boundary elements method to the calculation of reinforced concrete and fiber-reinforced concrete culverts is considered. The force state of the pipe as part of the road embankment from the action of the soil mass and moving vehicle load is determined by an alternating diagram of bending moments with a maximum positive moment in the ridge and trough sections and a maximum negative moment in the side sections, which necessitates the need for double rod reinforcement. Such reinforcement has low efficiency. It is difficult to ensure the design thickness of the protective layer of concrete, the displacement of the ring reinforcement in the direction of reducing the protective layer reduces the operational reliability of the structure in terms of durability, and with an increase in the thickness of the protective layer, the bearing capacity of the section decreases. One of the effective solutions to this problem is the use of dispersed steel fiber reinforcement, which allows you to avoid the use of double rod reinforcement. In addition, steel-fiber concrete improves the characteristics of concrete, and also allows you to change the nature of the destruction process. Unlike conventional concrete, in which this process occurs almost instantly, brittle destruction does not occur in fiber concrete, and the structure continues to resist the load, and the nature of the destruction changes from brittle to ductile. Literature analysis shows that in Ukraine, the emphasis in the study of culverts is on corrugated and plastic pipes, and very little attention is paid to reinforced concrete and fiber concrete pipes. In the work, the culvert is considered as a circular cylindrical shell. As a result of the implementation of the algorithm in SCILAB, the values of deflections, angles of rotation, bending moments, transverse forces and stresses were calculated. In order to verify the obtained results, computer modeling of the pipe and finite element analysis of its stress-strain state in the ANSYS program were performed. A comparison of the stressed and displaced values obtained by the two methods shows that the maximum difference between the values of the stressed values is 0.65%, and the displaced values are 0.51%, which indicates the effectiveness of the proposed calculation method.

Keywords: culvert, fiber-reinforced concrete, cylindrical shell, boundary element method, SCILAB, ANSYS.

РОЗРАХУНОК ВОДОПРОПУСКНОЇ ТРУБИ ЧИСЕЛЬНО-АНАЛІТИЧНИМ МЕТОДОМ ГРАНИЧНИХ ЕЛЕМЕНТІВ

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Анотація: Розглядається методика застосування чисельно-аналітичного методу граничних елементів до розрахунку залізобетонних та фібробетонних водопропускних труб. Силовий стан труби у складі насипу дороги від дії маси ґрунту і рухомого автомобільного навантаження визначається знакозмінною епюрою згинальних моментів з максимальним позитивним моментом в коньковому і лотковому перерізах і максимальним негативним моментом в бічних перерізах, що обумовлює необхідність подвійного стрижневого армування. Таке армування має низьку ефективність. Важко забезпечити проектну товщину захисного шару бетону, зміщення кільцевої арматури в бік зменшення захисного шару знижує експлуатаційну надійність конструкції по довговічності, а із збільшенням товщини захисного шару знижується несуча здатність перерізу. Одним з ефективних рішень цієї проблеми є застосування дисперсного армування сталевою фіброю, яке дозволяє уникнути використання подвійного стрижневого армування. Крім того, сталефібробетон покращує характеристики бетону, а також дозволяє змінити характер процесу руйнування. На відміну від звичайного бетону, в якому цей процес відбувається практично моментально, у фібробетоні не відбувається крихкого руйнування, і конструкція продовжує чинити опір навантаженню, а характер руйнування змінюється з крихкого на в'язкий. Аналіз літератури показує, що в Україні акцент у дослідженнях водопропускних труб робиться на гофрованих і пластмасових трубах, а залізобетонним і фібробетонним трубам приділяється дуже мало уваги. В роботі водопропускна труба розглядається, як кругова циліндрична оболонка. В результаті реалізації алгоритму у SCILAB обчислені значення прогинів, кутів повороту, згинальних моментів, поперечних сил і напружень. З метою верифікації одержаних результатів виконано комп'ютерне моделювання труби та скінченно-елементний аналіз її напружено-деформованого стану у програмі ANSYS. Порівняння напружень і переміщень, отриманих двома методами, показує, що максимальна розбіжність величин напружень становить 0,65%, а переміщень — 0,51%, що свідчить про ефективність запропонованого методу розрахунку.

Ключові слова: водопропускна труба, фібробетон, циліндрична оболонка, метод граничних елементів, SCILAB, ANSYS.

1 INTRODUCTION

Among many structures of circular cross-section, reinforced concrete culverts occupy a significant place. The force state of the pipe as part of the road embankment from the action of the soil mass and moving vehicle load is determined by an alternating diagram of bending moments with a maximum positive moment in the ridge and trough sections and a maximum negative moment in the side sections, which necessitates the need for double rod reinforcement.

The low efficiency of such reinforcement is due to the fact that in this case the relative working height of the section is small due to the need to ensure the requirements for the thickness of the protective layer of concrete for the working reinforcement. In this case, it is difficult to ensure the design thickness of the protective layer of concrete, the displacement of the circular reinforcement in the direction of reducing the protective layer reduces the operational reliability of the structure in terms of durability, and with an increase in the thickness of the protective layer, the bearing capacity of the section decreases. One of the effective solutions to this problem is the use of dispersed steel fiber reinforcement, which allows you to avoid the use of double rod reinforcement. In addition, steel fiber concrete allows you to improve such characteristics of concrete as crack resistance, frost resistance, tensile strength, bending, torsion, etc., and also allows you to change the nature of the destruction process. Unlike conventional concrete, in which this process occurs almost instantly, brittle destruction does not occur in fiber concrete, and the structure continues to resist the load, and the nature of the destruction changes from brittle to ductile.

2 ANALYSIS OF LITERATURY DATA AND RESOLVING THE PROBLEM

Literature analysis shows that in Ukraine, the emphasis in the study of culverts is on corrugated and plastic pipes, and very little attention is paid to reinforced concrete and fiber concrete pipes. We note the works [1-3]. The monograph [4] is very informative and useful in terms of the study of fiber concrete pipes. In work [1], the stress-strain state of fiber concrete water and sewer pipes manufactured by the dry vibropressing method was considered. Laboratory tests of fiber concrete samples for compression, bending, crack resistance, tension and heating were carried out at the test site. The optimal amount of fiber in concrete and the necessary calculated mechanical characteristics of fiber concrete were determined. The authors of the article [2] provide interesting statistics: as of the beginning of 2019, there were about 130 thousand pieces of culverts on public roads in Ukraine, more than 80% of which are reinforced concrete pipes.

It is known that the low tensile strength and brittleness of concrete [5-7] reduce its structural efficiency. These disadvantages can be significantly compensated by dispersed reinforcement of concrete [8, 9]. The resulting fiber-reinforced concrete is advantageously different from traditional concrete, having several times higher tensile and shear strength, impact strength, crack resistance, abrasion, frost resistance, water resistance, cavitation resistance, fatigue strength, heat resistance, fire resistance, fracture toughness. Studies have shown that the most indicative characteristics of fiber-reinforced concrete are strength, toughness and fracture work in axial tension and bending. According to the latter indicator, fiber-reinforced concrete can exceed standard concrete by 15-20 times [7-9].

From the point of view of structural mechanics, a culvert, regardless of the material used, is a long cylindrical shell. Several methods have been recommended for analyzing the operation of cylindrical shells. Simplified equations derived by Shorer [10] can be used. A more complete system of equations, which, however, requires a lot of computational work, was formulated by Jenkins [11]. The main analytical methods for their calculation were developed in the last century. These are the moment-free and moment theories of shells, and the semi-moment theory [12]. The issues of strength and crack resistance of reinforced concrete cylindrical shells have long attracted the attention of scientists [13]. This interest does not wane even now. We note the works [14-17].

The analysis of the literature shows that very little attention is paid to the development of new methods for calculating reinforced concrete and steel-fiber concrete cylindrical shells, so this direction is relevant.

3 PURPOSE AND TASKS OF THE STADY

The purpose of the work is the application of the numerical-analytical boundary elements method to the calculation of reinforced concrete and fiber-reinforced concrete culverts.

To achieve the set purpose, it is necessary to apply the numerical-analytical boundary elements method to the calculation of a culvert, which is considered as a circular cylindrical shell, to perform a numerical analysis of the pipe using the finite element method and to conduct a comparative analysis of the results obtained by the two methods.

4 MATERIALS AND METHODS OF RESEARCH

Reinforced concrete and fiber concrete cylindrical shells are considered. Computer modeling methods, methods of higher mathematics, numerical-analytical boundary element method, finite element method are used.

5 RESEARCH RESULTS

Consider a culvert (Fig. 1), which from the point of view of structural mechanics is a closed circular cylindrical shell of constant thickness. We will assume that the shell is rigidly clamped at the ends, rests on two intermediate supports and is under the action of uniform external pressure. The beam model of such a shell is presented in Fig. 2.

Fig. 1. Culvert

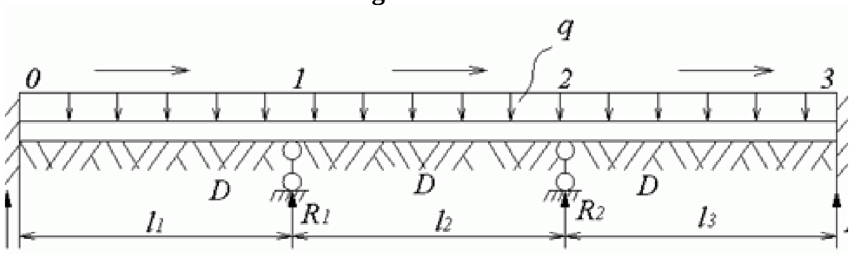


Fig. 2. Closed circular cylindrical shell loaded with uniformly distributed pressure

A detailed description of the numerical-analytical boundary element method and examples of its implementation can be found in [18].

The algorithm for calculating a cylindrical shell is as follows:

- we break the shell (replacing its beam) into several modules. In this case, since it is assumed that the pipe rests on two intermediate supports, there will be three such modules;
- we form matrices of initial and final parameters and the load vector, taking into account the boundary conditions, equilibrium equations and compatibility equations of the displacements of nodes 1 and 2.

Each module as a one-dimensional physical body has only two boundary points − $x=0$ and $x=l$. If the coordinate x of each rod is given the boundary value l_i, then a sufficiently simple transformation can be performed according to the scheme

$$\vec{Y}(l)=\bar{A}(l)\vec{X}(0)+\vec{B}(l)\rightarrow\bar{A}(l)\vec{X}(0)-\vec{Y}(l)=-\vec{B}(l)\rightarrow\bar{A}_*(l)\vec{X}_*(0,l)=-\vec{B}(l), \quad (1)$$

where the finite boundary parameters of the matrix $\vec{Y}$ are transferred to the place of the zero parameters of the vector $\vec{X}$. In this case, these vectors are supplemented with equations of equilibrium and commonality of displacements of nodal points and boundary conditions. At the end of the transformation scheme (1) we have a system of linear equations for the initial and final parameters of all modules of the structure. After calculating the initial parameters of the modules, their stress-strain state is determined by the matrix equation

$$\vec{Y}(x)=\bar{A}(x)\vec{X}(0)+\vec{B}(x). \quad (2)$$

Thus, the solution of direct problems of structural mechanics of rod systems in the NA BEM is reduced to the solution of one system of linear algebraic equations and the calculation of the stress-strain state at the internal points of the rods according to the ratio of the initial parameter method. Such a solution scheme ensures obtaining very accurate results.

The main operation in scheme (1) is the transfer of parameters from $\vec{Y}$ to $\vec{X}$. The process of transferring the final parameters of a vector $\vec{Y}$ to a vector $\vec{X}$ is based on the following provisions. Vectors of any rod (and non-rod) structure at the limit value of the coordinate $x=l_i$ will contain 3 groups of parameters. The first group is the zero boundary parameters, determined by the given support conditions (boundary conditions). The second group is the dependent parameters, related to each other by the equations of equilibrium and the commonality of displacements of the structural nodes. The third group of boundary parameters is not related to each other. These parameters can be conditionally called independent. Transferring parameters from vector $\vec{Y}$ to vector $\vec{X}$ must be compensated by non-zero elements of the matrix $\bar{A}$, otherwise the original equation of the scheme (1) is violated.

It is obvious that the independent parameters of the vector $\vec{Y}$ must be transferred to the place of the zero parameters of the vector $\vec{X}$, and the dependent parameters are transferred according to the equations of their connection.

Before the parameter transfer operation, it is necessary to free the matrix $\bar{A}$ fields from elements associated with zero parameters of the vector $\vec{X}$, i.e., to zero the columns of the matrix $\bar{A}$, the numbers of which are equal to the numbers of zero rows of the vector $\vec{X}$. Next, non-zero compensating elements are introduced into the matrix $\bar{A}$ and the transformations according to scheme (1) are completed, since only the signs of the elements change in the matrix $\vec{B}$ [18].

We write the equation of the boundary value problem in the form

$$\bar{A}\vec{X}_* = \vec{B}, \tag{3}$$

where

$$\vec{X}_* = \begin{vmatrix} Q^{0-1}(l_1) \\ Q^{1-2}(l_2) \\ M^{0-1}(0) \\ ;Q^{0-1}(0) \\ M^{2-3}(l_3) \\ D\phi^{1-2}(0) \\ M^{1-2}(0) \\ Q^{1-2}(0) \\ Q^{2-3}(l_3) \\ D\phi^{1-2}(0) \\ M^{1-2}(0) \\ Q^{1-2}(0) \end{vmatrix}; \quad \vec{B} = \begin{vmatrix} B_{11}^{0-1}(l_1) \\ B_{21}^{0-1}(l_1) \\ B_{31}^{0-1}(l_1) \\ B_{41}^{0-1}(l_1) \\ B_{41}^{1-2}(l_2) \\ B_{21}^{1-2}(l_2) \\ B_{31}^{1-2}(l_2) \\ B_{41}^{1-2}(l_2) \\ B_{11}^{2-3}(l_3) \\ B_{21}^{2-3}(l_3) \\ B_{31}^{2-3}(l_3) \\ B_{41}^{2-3}(l_3) \end{vmatrix}.$$

The matrix $\bar{A}$ of fundamental functions in (1) after the above transformations has the form

$$\bar{A} = \begin{vmatrix} 0 & 0 & -A_{13} & -A_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -A_{23} & -A_{24} & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & A_{34} & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & A_{43} & A_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{12} & -A_{13} & -A_{14} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{22} & -A_{23} & -A_{24} & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -A_{32} & A_{33} & A_{34} & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -A_{42} & A_{43} & A_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{12} & -A_{13} & -A_{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}.$$

Analytical expressions of fundamental functions (components of this matrix) are obtained in [18] for all possible roots of the characteristic equation corresponding to the differential equation of the problem.

Solving system (3), we obtain the kinematic and static parameters of the stress-strain state of the shell.

6 DISCUSSION OF RESEARCH FINDINGS

In accordance with the given algorithm, the calculation of the fiber concrete culvert under the action of uniform external pressure was performed (Fig. 3).

As a result of the implementation of the algorithm in SCILAB [19], the values of deflections, angles of rotation, bending moments, transverse forces and stresses were calculated.

Numerical values of deflections and tension, calculated with a step of 1 m at the points of the upper forming shell, are given in the table. 1.

The finite-element model of the pipe is built in the ANSYS program [20].

Here, as in SCILAB, the values of deflections, angles of rotation, bending moments, transverse forces and stresses are calculated (Table 1). The tension and displacement diagrams are shown in fig. 4.

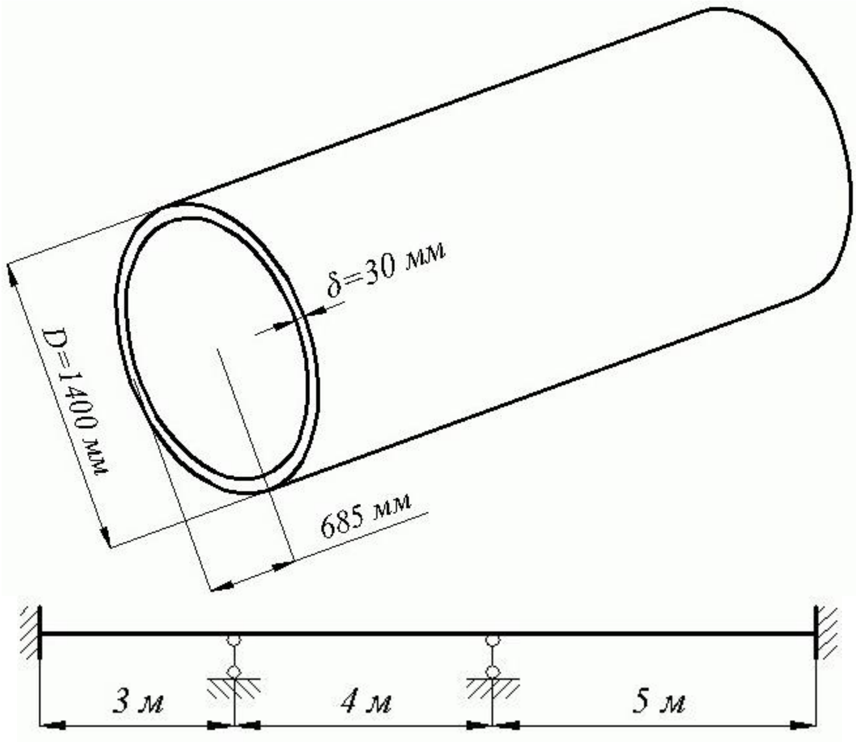
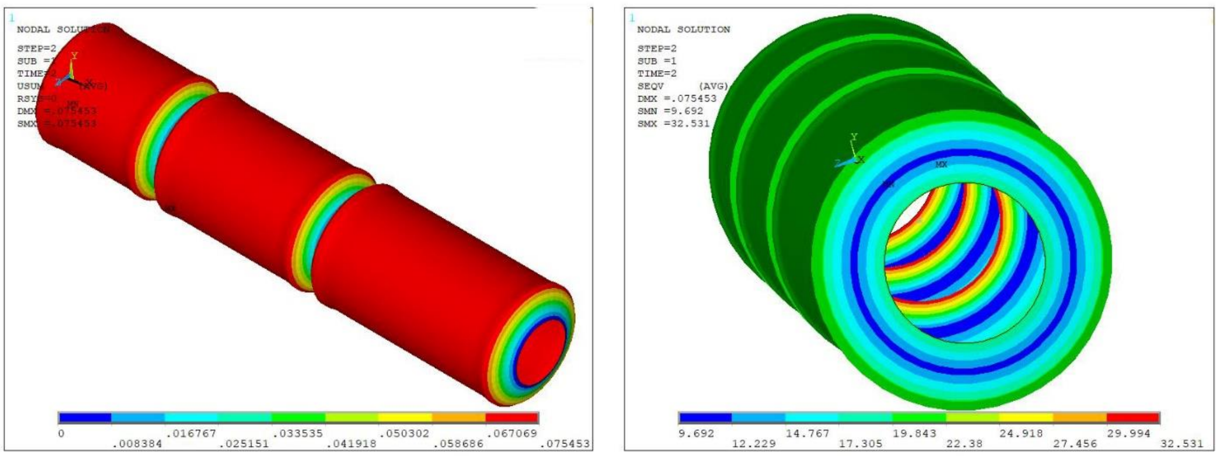


Fig. 3. Calculation scheme of a water culvert



a b

Fig. 4. Stresses (a) and displacements (b) in the water culvert

Table 1

Stresses and displacements in the fiber concrete water culvert

Coordinate along the axis, m	МГЭ, SCILAB		МКЭ, ANSYS	
	Stresses, MPa	Displacement, mm	Stresses, MPa	Displacement, mm
1	20,871	0,7133	20,796	0,7132
2	20,644	0,6803	20,796	0,6830
3	32,743	0,0164	32,530	0,0162
4	20,643	0,7103	20,796	0,7139
5	20,675	0,7137	20,792	0,7130
6	20,912	0,7139	20,796	0,7130
7	32,743	0,0164	32,530	0,0162
8	20,716	0,7131	20,796	0,7140
9	20,715	0,7101	20,792	0,7130
10	20,715	0,7108	20,792	0,7130
11	20,779	0,7135	20,796	0,7139

7 CONCLUSIONS

Thus, the method of applying the numerical analytical method of boundary elements to the calculation of reinforced concrete and fiber concrete culverts is proposed. In accordance with the given algorithm, the calculation of the culvert for the action of external pressure was performed. As a result of the implementation of the algorithm in SCILAB, the values of deflections, angles of rotation, bending moments, transverse forces and stresses were calculated. In order to verify the obtained results, computer modeling of the pipe and finite-element analysis of its stress-strain state were performed in the ANSYS program. A comparison of the stressed and displaced values obtained by the two methods shows that the maximum discrepancy between the stressed and displaced values is 0.65%, and the displaced value is 0.51%, which indicates the effectiveness of the proposed calculation method.

8 ETHICAL DECLARATIONS

The authors have no relevant financial or non-financial interests to report.

References

1. Mastanzade N.S., Rasulov H.I., Rustamli T.M., Altun F. Research in stress-strain state of underground fiber-concrete pipes. Construction Materials. 2019. No. 10. Pp. 16–21. DOI: https://doi.org/10.31659/0585-430X-2019-775-10-16-21
2. Kaskiv V.I. Deformations determined by the conditions of static operation of the pipe in the "embankment - pipe - base" structure / V. I. Kaskiv, O. S. Chechuga, O. P. Lysenko, A. M. Lysenko // Highways and road construction. Scientific and technical collection. K.: NTU, 2012. Issue. 82. P.173 – 180. [in Ukrainian].
3. Calculation and construction of reinforced concrete culverts: a textbook / V.K. Zhdanyuk, V.P. Kozhushko, O.G. Kislov, V.S. Tytar. Kharkiv: KhNADU, 2011. 200 p. [in Ukrainian].
4. Andriychuk O.V. Steel-fiber concrete pressureless pipes: monograph / O.V. Andriychuk, E.M. Babych. Lutsk: RVV Lutsk NTU, 2012. 150 p. [in Ukrainian].
5. Doroshenko Yu., Doroshenko O., Chyzhenko N., Gudymenko K. Dispersion-reinforced concrete – a reliable and effective material for transport construction // Transport Construction of Ukraine. 2007. No. 121 (5) Pp. 16 – 19. [in Ukrainian].
6. Drobishynets S. Ya. Research on the mechanical characteristics of steel fiber concrete during

repeated compression and tension // Prospects for the development of building structures, buildings, structures and their foundations: Interdepartmental scientific and technical collection. Kyiv: NIIBK, 2003. Issue 58: Pp. 39 – 45. [in Ukrainian].
7. Peng, Shilong & Rong, Chuanxin & Cheng, Hua & Wang, Xiaojian & Li, Mingjing & Tang, Bin & Li, Xuemei. (2019). Mechanical Properties of High-Strength High-Performance Reinforced Concrete Shaft Lining Structures in Deep Freezing Wells. Advances in Civil Engineering. 2019. 10.1155/2019/2430652.
8. Kwon S-H, Ferron R.P., Akkaya Y., Shah S.P. Cracking of fiber-reinforced self-compacting concrete due to restrained shrinkage. //International journal of concrete structures and materials, vol.1, pp. 3-9, Dec. 2007.
9. Yusuf M., Isak A., Mohamud G., Warsame A., Osman Y., Ibrahim A. and Elmi L. (2023) Effect of Steel Fiber on Concrete's Compressive Strength. Open Journal of Civil Engineering, vol 13, Pages 192-197, ISSN 2164-3172, https://doi.org/10.4236/ojce.2023.131014.
10. Schorer, H. (1935): Line load action on thin cylindrical shells, P r o c . ASCE. 281 p.
11. Jenkins, R. S. (1947): 'Theory and Design of Cylindrical Shell Structures'O. V. Arup, London.
12. Krutii Yu.S. Methods of calculating cylindrical shells. Monograph/ Krutii Yu.S., Surianinov M.G., Chuchmai O.M.// Odesa: ODABA, 2018, 182 p. [in Ukrainian].
13. Kan S.N., Kaplan Y.I., Calculation of cylindrical shells of building coverings. Year: 1973. Publisher: Higher School 244 p. [in Ukrainian].
14. Tanaka, Masafumi & Mashita, Kazuhiko. (2010). Retrofitted strength of concrete cylindrical shells reinforced with single layer under concentrated load. Journal of Structural and Construction Engineering (Transactions of AIJ). 75. 1681-1689. 10.3130/aijs.75.1681.
15. Karpov, Vladimir & Semenov, Alexey. (2013). Mathematical model of deformation of orthotropic reinforced shells of revolution. Magazine of Civil Engineering. 40. 100-106. 10.5862/MCE.40.11.
16. Shen Li, Do Kyun Kim, Qing Quan Liang. Fibre-Based modelling for predicting the progressive collapse of cylindrical shells under combined axial compression and bending moment, Engineering Structures, Volume 272, 2022, 114988, ISSN 0141-0296, doi.org/10.1016/j.engstruct.2022.114988.
17. Rong Li, Meng Yang, Bin Liang. A new and convenient method for strength evaluation of cracked cylindrical shell based on the ratio of crack tip stresses. Structures, Volume 52, 2023, Pages 146-157, ISSN 2352-0124, https://doi.org/10.1016/j.istruc.2023.03.130.
18. Dashchenko A.F., Kolomiets L.V., Orobey V.F., Surianinov M.G. Numerical-analytical method of boundary elements. Odesa: VMV, 2010. In 2 volumes. Vol. 1. 416 p. Vol. 2. 512 p. [in Ukrainian].
19. Fetisov V. S. Mathematical system Scilab: teaching-methodical manual. 2nd ed., revised and supplemented. Nizhyn: NDU named after M. Gogol, 2022. 82 p. [in Ukrainian].
20. Lazarieva D.V. Techniques for working with the ANSYS PC when solving problems in mechanics. Edited by M.G. Surianinov: monograph/ Lazareva D.V., Soroka M.M., Shilyaev O.S., Suryaninov M.G.// Odesa: ODABA, 2020. 432 p.[in Ukrainian].

Література

1. Mastanzade N.S., Rasulov H.I., Rustamli T.M., Altun F. Research in stress-strain state of underground fiber-concrete pipes. Construction Materials. 2019. No. 10. Pp. 16–21. DOI: https://doi.org/10.31659/0585-430X-2019-775-10-16-21
2. Каськів В.І. Деформації, які визначаються умовами статичної роботи труби в конструкції "насип – труба – основа" / В. І. Каськів, О. С. Чечуга, О. П. Лисенко, А. М. Лисенко // Автомобільні дороги і дорожнє будівництво. Науково-технічний збірник. К.: НТУ, 2012. Вип. 82. С.173 – 180.
3. Розрахунок та будівництво залізобетонних водопропускних труб: навчальний посібник / В.К. Жданюк, В.П. Кожушко, О.Г. Кіслов, В.С. Титар. Харків: ХНАДУ, 2011. 200 с.
4. Андрійчук О.В. Сталефібробетонні безнапірні труби: монографія / О.В. Андрійчук, Є.М. Бабич. Луцьк: РВВ Луцького НТУ, 2012. 150 с.
5. Дорошенко Ю., Дорошенко О., Чиженко Н., Гудименко К. Дисперсно-армований бетон – надійний та ефективний матеріал для транспортного будівництва // Транспортное

строительство Украины. 2007. № 121 (5) С. 16 – 19.
6. Дробишинець С. Я. Дослідження механічних характеристик сталефібробетону при повторному стисненні та розтягненні // Перспективи розвитку будівельних конструкцій, будівель, споруд та їх основ: Міжвідомчий науково-технічний збірник. К.: НДІБК, 2003. Вип. 58: С. 39 – 45.
7. Peng, Shilong & Rong, Chuanxin & Cheng, Hua & Wang, Xiaojian & Li, Mingjing & Tang, Bin & Li, Xuemei. (2019). Mechanical Properties of High-Strength High-Performance Reinforced Concrete Shaft Lining Structures in Deep Freezing Wells. Advances in Civil Engineering. 2019. 10.1155/2019/2430652.
8. Kwon S-H, Ferron R.P., Akkaya Y., Shah S.P. Cracking of fiber-reinforced self-compacting concrete due to restrained shrinkage. //International journal of concrete structures and materials, vol.1, pp. 3-9, Dec. 2007.
9. Yusuf M., Isak A., Mohamud G., Warsame A., Osman Y., Ibrahim A. and Elmi L. (2023) Effect of Steel Fiber on Concrete's Compressive Strength. Open Journal of Civil Engineering, vol 13, Pages 192-197, ISSN 2164-3172, https://doi.org/10.4236/ojce.2023.131014.
10. Schorer, H. (1935): Line load action on thin cylindrical shells, P r o c . ASCE. 281 p.
11. Jenkins, R. S. (1947): 'Theory and Design of Cylindrical Shell Structures'O. V. Arup, London.
12. Крутій Ю.С. Методи розрахунку циліндричних оболонок. Монографія/ Крутій Ю.С., Сур'янінов М.Г., Чучмай О.М.// Одеса: ОДАБА, 2018, 182 с.
13. Кан С.Н., Каплан Ю.І., Розрахунок циліндричних оболонок покриттів будівель. Рік: 1973р. Видавництво: Вища школа 244 с.
14. Tanaka, Masafumi & Mashita, Kazuhiko. (2010). Retrofitted strength of concrete cylindrical shells reinforced with single layer under concentrated load. Journal of Structural and Construction Engineering (Transactions of AIJ). 75. 1681-1689. 10.3130/aijs.75.1681.
15. Karpov, Vladimir & Semenov, Alexey. (2013). Mathematical model of deformation of orthotropic reinforced shells of revolution. Magazine of Civil Engineering. 40. 100-106. 10.5862/MCE.40.11.
16. Shen Li, Do Kyun Kim, Qing Quan Liang. Fibre-Based modelling for predicting the progressive collapse of cylindrical shells under combined axial compression and bending moment, Engineering Structures, Volume 272, 2022, 114988, ISSN 0141-0296, doi.org/10.1016/j.engstruct.2022.114988.
17. Rong Li, Meng Yang, Bin Liang. A new and convenient method for strength evaluation of cracked cylindrical shell based on the ratio of crack tip stresses. Structures, Volume 52, 2023, Pages 146-157, ISSN 2352-0124, https://doi.org/10.1016/j.istruc.2023.03.130.
18. Дащенко А.Ф., Коломієць Л.В., Оробей В.Ф., Сур'янінов М.Г. Чисельно-аналітичний метод граничних елементів. Одеса: ВМВ, 2010. У 2-х томах. Т.1. 416 с. Т.2. 512 с.
19. Фетісов В. С. Математична система Scilab: навч.-метод. посібн. 2-е вид., переробл. і доп. Ніжин: НДУ ім. М. Гоголя, 2022. 82 с.
20. Лазарєва Д.В. Прийоми роботи з ПК ANSYS при розв'язанні задач механіки. Під редакцією М.Г. Сур'янінова: монографія/ Лазарєва Д.В., Сорока М.М., Шиляєв О.С., Сур'янінов М.Г.// Одеса: ОДАБА, 2020. 432 с.

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