UDC 614.841

IMPROVED THERMOMECHANICAL MODEL FOR PREDICTING THE BEHAVIOUR OF REINFORCED CONCRETE STRUCTURES UNDER FIRE AND EXPLOSION CONDITIONS

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Abstract: The article presents the results of a study aimed at improving the thermomechanical model for predicting the behaviour of monolithic reinforced concrete structures when exposed to elevated fire temperatures and overpressure from a deflagration explosion. The relevance of the topic is due to the growing requirements for the reliability of modern buildings, especially critical infrastructure facilities, where the combination of thermal and dynamic loads creates particularly dangerous operating conditions. The model is implemented using the finite element method, which makes it possible to comprehensively take into account transient heat transfer, temperature-dependent material properties, and impulsive loads from explosions, described in the form of triangular and exponential functions. The dynamic behaviour of reinforced concrete structures is represented by the equation of motion, considering temperature-dependent stiffness, Rayleigh damping, and thermal deformations of both concrete and reinforcement.

A comparative analysis of code-specified material properties and the results of full-scale tests was carried out. Significant differences were found in the curves describing the reduction in strength and initial modulus of elasticity when concrete is heated, as well as in the strength values of reinforcement of different grades. Based on experimental data, analytical and polynomial relationships were proposed that reproduce the actual behaviour of heavy concrete and lightweight expanded clay aggregate concrete at elevated temperatures, along with formulas for determining reduction factors for reinforcement strength. This ensures a significant improvement in the accuracy of predictions of the load-bearing capacity and deformability of reinforced concrete structures. The model can be used as a tool in methodologies for assessing the robustness of monolithic reinforced concrete buildings against progressive collapse caused by fire and explosion. At the same time, the limitations of the model are outlined: it does not take into account crack formation, creep, or the spatial effects of blast wave propagation. Directions for further research are identified, aimed at expanding the practical applicability of the model and improving methods for assessing the structural stability of buildings.

Keywords: monolithic reinforced concrete buildings; fire; deflagration explosion; finite element method; temperature-dependent properties; reinforcement resistance reduction coefficients, progressive collapse.

УДОСКОНАЛЕНА ТЕРМОМЕХАНІЧНА МОДЕЛЬ ДЛЯ ПРОГНОЗУВАННЯ ПОВЕДІНКИ ЗАЛІЗОБЕТОННИХ КОНСТРУКЦІЙ В УМОВАХ ПОЖЕЖІ ТА ВИБУХУ

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Анотація: У статті представлено результати дослідження, спрямованого на удосконалення термомеханічної моделі для прогнозування поведінки монолітних залізобетонних конструкцій за умов дії підвищених температур пожежі та надлишкового тиску від дефлаграційного вибуху. Актуальність теми зумовлена зростанням вимог до надійності сучасних будівель, особливо об’єктів критичної інфраструктури, де поєднання теплових і динамічних навантажень створює особливо небезпечні умови експлуатації. Модель реалізовано із застосуванням методу скінченних елементів, що дозволяє комплексно враховувати нестаціонарний теплообмін, температурно-залежні характеристики матеріалів та імпульсні навантаження від вибухів, описані у вигляді трикутних і експоненційних функцій. Динамічна поведінка залізобетонних конструкцій подається рівнянням руху з урахуванням температурно-залежної жорсткості, демпфування за Релеєм та термічних деформацій як бетону, так і арматури.

Проведено порівняльний аналіз нормативних характеристик матеріалів та результатів натурних випробувань. Виявлено істотні розбіжності у кривих, що описують зниження міцності та початкового модуля пружності бетону при нагріванні, а також у значеннях міцності арматури різних класів. На основі експериментальних даних запропоновано аналітичні та поліноміальні залежності, які відтворюють фактичну поведінку важкого бетону та бетону на легкому заповнювачі (керамзитобетону) за підвищених температур, а також формули для визначення коефіцієнтів зниження міцності арматури. Це забезпечує суттєве підвищення точності прогнозування несучої здатності та деформативності залізобетонних конструкцій. Розроблена модель може використовуватися як інструмент у методиках оцінювання живучості монолітних залізобетонних будівель щодо прогресуючого руйнування, спричиненого пожежею та вибухом. Водночас окреслено обмеження моделі: не враховується утворення тріщин, повзучість та просторові ефекти поширення вибухової хвилі. Визначено напрями подальших досліджень, орієнтованих на розширення практичної застосовності моделі та вдосконалення методів оцінювання конструктивної стійкості будівель.

Ключові слова: монолітні залізобетонні будівлі; пожежа; дефлаграційний вибух; метод скінченних елементів; температурно-залежні характеристики; коефіцієнти зниження міцності арматури; прогресуюче руйнування.

1 INTRODUCTION

Modern monolithic reinforced concrete buildings are widely used in civil and industrial construction due to their high operational properties, versatility, and cost-effectiveness. However, in emergency situations such as fires and explosions, these structures are subjected to significant thermal and dynamic effects, which reduce their load-bearing capacity and deformability and may lead to progressive collapse. Ensuring an adequate level of reliability and robustness of building structures under such conditions is one of the key challenges of modern construction.

Existing design codes [1, 2] provide relationships for accounting for the influence of elevated temperatures on the strength and deformability of concrete and reinforcement. However, the results of full-scale tests show significant discrepancies between the code-specified curves and the actual behaviour of materials, which may reduce the accuracy of predicting the performance of structures under high-temperature and explosive loading. As a result, there is a need to improve analytical models that incorporate the changes in thermophysical and strength properties of materials obtained from experimental tests, as well as the dynamic character of explosive loading.

One promising approach is the application of thermomechanical models within the finite element method framework. Such models enable a comprehensive account of heat transfer processes, temperature-dependent material properties, thermal expansion effects, and the dynamic nature of loads. Enhancing these models provides a basis for improving the accuracy of evaluating the structural stability of reinforced concrete structures and reducing the risk of progressive collapse.

2 ANALYSIS OF LITERARY DATA AND RESOLVING THE PROBLEM

Monolithic reinforced concrete structures are widely used in civil and industrial construction due to their combination of high strength, durability, constructability, and cost-effectiveness. They are employed in the construction of buildings and facilities of various types, ranging from residential complexes to critical infrastructure. However, evidence from accidents and disasters indicates that even such structures remain vulnerable when exposed to extreme factors. The most hazardous among these are fires and explosions, which can cause a significant reduction in load-bearing capacity, loss of stability, and, in critical cases, structural progressive collapse [1–3].

The effect of high temperatures on concrete and reinforcement has been the subject of extensive research over many years. Scientific works by Khoury G. A. [1], Lie T. T. [2], Milovanov A. F. [3], and others present experimental relationships that describe changes in the strength, modulus of elasticity, and deformability of materials under heating. In most countries, design practice relies on the provisions of standards EN 1992–1–1, EN 1992–1–2, and the national standard DBN V.2.6-98, which provide generalized curves for considering the influence of temperature on the properties of concrete and reinforcement. However, numerous experimental studies demonstrate that the actual behaviour of materials differs significantly from these normative relationships. This discrepancy is explained by variations in concrete composition, type of aggregates, moisture content, curing conditions, and service environment [1–4]. As a result, calculations based solely on normative curves may not always ensure sufficient accuracy in predicting the behaviour of structures exposed to fire.

The issue of blast resistance of structures is also actively studied in the international literature. Numerous works are devoted to modelling the impact of shock waves on building systems, determining explosion pressure, its duration, and the consequences for load-bearing elements. However, most of these studies focus on the isolated analysis of explosive loads without considering the prior heating of structures. At the same time, practical experience from real accidents indicates that fire and explosion often occur in combination (for example, at industrial and energy facilities), creating particularly hazardous operating conditions [4–6]. Special attention in recent research has been paid to the phenomenon of progressive collapse, in which the local failure of a single element triggers a cascade of failures throughout the entire structural system. The works of Otrosh Y. and Maiboroda R. [4–6] have demonstrated that, in order to prevent this phenomenon, there is a need for models capable of adequately capturing the combined effects of elevated temperatures and dynamic loads.

One promising approach is the use of thermomechanical models within the finite element method. Such models allow for comprehensive consideration of non-stationary heat transfer, temperature-dependent material properties, thermal expansion effects, and impulse explosive loads. However, most existing solutions are simplified and do not fully incorporate experimental data, which limits the accuracy of the assessment of structural stability. In addition, integrated models that simultaneously reproduce the combined effects of fire and explosion remain insufficiently investigated, and the available results require systematization and verification.

Thus, the analysis of the literature confirms the relevance of improving the accuracy of predicting the stability of reinforced concrete structures under combined fire and explosion effects. The research problem consists in developing an improved thermomechanical model based on the results of field tests and regulatory provisions, which would reduce uncertainty in predictions and thereby enhance the reliability and safety of modern building structures.

3 PURPOSE AND TASKS OF THE STADY

The aim of the study is to develop and substantiate an improved thermomechanical model for predicting the behaviour of reinforced concrete structures under fire and explosion conditions, taking into account experimental data and design code provisions.

To achieve this aim, the following research tasks have been defined:

1. Identify and describe the components of a mathematical model of the fire resistance of building structures. Compare the characteristics of concrete and reinforcement at elevated temperatures with code-specified curves and the results of full-scale tests.

2. Propose analytical relationships to describe the reduction in strength and initial modulus of elasticity of concrete, as well as reduction factors for the strength of reinforcement of different grades at elevated temperatures.

3. Develop an improved thermomechanical model within the finite element method framework, taking into account heat transfer, thermal expansion of materials, and the dynamic nature of explosive loading.

4 BASIC RESULTS

To assess the stability of monolithic reinforced concrete buildings as a result of fire and deflagration explosion, a mathematical model has been improved, which allows calculations to be performed for the structural system of the entire building, taking into account contact heat exchange between structures and dynamic loads from the explosion.

Thermal effects.

Thermal effects are expressed as absorbed heat flux $\dot{h}_{net}$ Wt/m^2 on the surface of the structure, in accordance with clause 3.1 of EN 1991–1–2. On the heated surface, the absorbed heat flux $\dot{h}_{net}$ must be determined taking into account convective and radiative heat transfer

$$\dot{h}_{net} = \dot{h}_{net,c} + \dot{h}_{net,r}, \tag{1}$$

where: $\dot{h}_{net,c}$ – convective component of absorbed heat flux, Wt/m^2; $\dot{h}_{net,r}$ – radiative component of absorbed heat flux, Wt/m^2.

The convective component of absorbed heat flux is determined by the formula:

$$\dot{h}_{net,c} = \alpha_c \left(\Theta_g - \Theta_m\right), \quad (2)$$

where α_c – convective heat transfer coefficient, Wt/m^2; Θ_g – temperature of the gas environment adjacent to the structure exposed to fireв, °C; Θ_m – of the structure surface, °C.

The absorbed heat flux $\dot{h}_{net}$ on the non-heated surface of structures should be determined using formula (2), where $\alpha_c = 4Wt/m^2$. The convective heat transfer coefficient $\alpha_c = 9Wt/m^2$ provided that the effects of radiative heat transfer are taken into account.

The radiation component of the absorbed heat flux per unit of surface area is determined by the formula

$$\dot{h}_{net,r} = \Phi\varepsilon_m\varepsilon_f\sigma\left[\left(\Theta_r + 273\right)^4 - \left(\Theta_m + 273\right)^4\right], \quad (3)$$

where Φ – angular coefficient; ε_m – degree of blackness of the structure surface; ε_f – emissivity of the flame; σ – Stefan-Boltzmann constant ($5,67\cdot10^{-8}Wt/m^2K^{-4}$); Θ_r – effective radiation temperature of the fire environment, °C; the emissivity of fire is usually taken as $\varepsilon_f = 1,0$.

In cases where structures are exposed to fire on all sides, the radiation temperature Θ_r can be represented by the temperature of the gas environment Θ_g around the structure. The surface temperature Θ_m is determined by the thermal calculation of the structure in accordance with Part 1-2 in the fire resistance calculation of EN 1992, EN 1996 and EN 1999, respectively. The temperature of the gas environments Θ_g can be taken as the nominal temperature conditions in accordance with clause 3.2 of EN 1991–1–2 or in accordance with the fire models given in clause 3.3 of EN 1991–1–2.

Thermal characteristics of concrete.

When calculating the fire resistance of reinforced concrete structures, a distinction is made between concretes based on silicate (granite, syenite, diorite) and limestone (limestone containing at least 80% of the weight of the concrete limestone component) aggregates. The nature of the change in the specific heat capacity of concrete $c_p\left(\theta\right), kJ/(kgK)$ is shown in Figure 1.

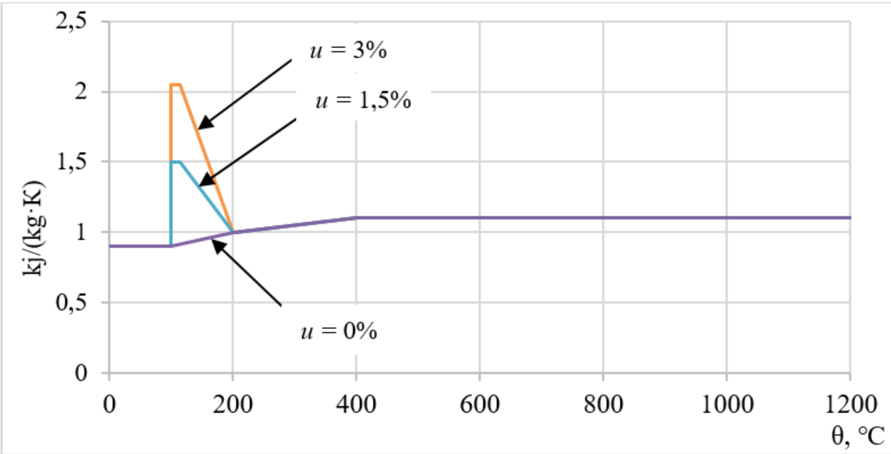

Fig. 1. Dependence of the specific heat capacity $c_\rho\left(\theta\right)$ of concrete on silicate aggregate with moisture content u = 0; 1.5 and 3 % on temperature

The thermal conductivity of concrete λ_c is determined within the range between the lower and upper limit values (Figure 2). The thermal conductivity value can be set within the range defined by the lower and upper limits.

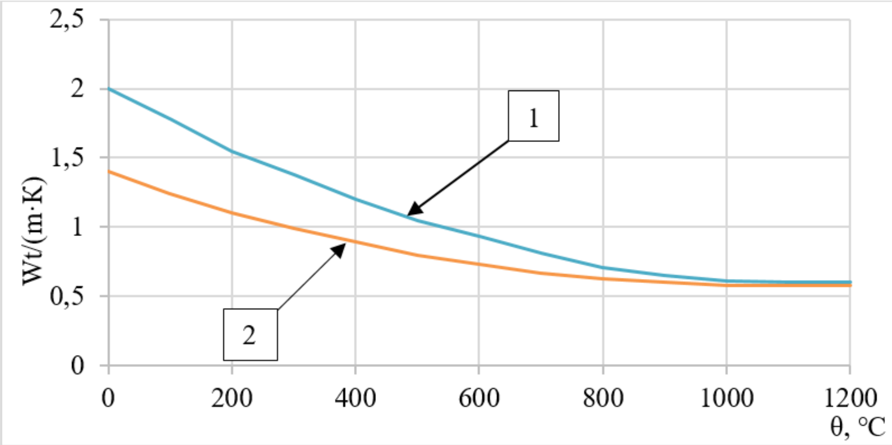

Fig. 2. Dependence of concrete thermal conductivity λ_c on temperature: 1 – upper limit; 2 – lower limit

In general, when the model parameters depend on temperature and radiation occurs, equation (4) is non-stationary and non-linear. It is solved using the finite element method.

Nominal temperature conditions.

The standard temperature regime is the nominal temperature regime defined in Eurocode 1 to represent a fully developed fire in a fire compartment. It is determined by the formula:

$$\Theta g = 20 + 345\, lg(8t + 1)\ . \qquad (4)$$

The convective heat transfer coefficient is $\alpha_c = 25Wt/m^2$.

The temperature regime of an external fire is the nominal temperature regime designed for the external surface of external enclosing walls that may be exposed to fire from different parts of the facade, i.e. directly from inside the corresponding fire compartment or compartment located below or adjacent to the corresponding external wall. It is determined by the formula:

$$\Theta_g = 660\left(1 - 0,687\,e^{-0,32t} - 0,313\,e^{-3,8t}\right) + 20. \qquad (5)$$

The convective heat transfer coefficient is $\alpha_c = 25Wt/m^2$.

The hydrocarbon temperature regime is the nominal temperature regime that shows the effects of a hydrocarbon fire. It is determined by the formula:

$$\Theta_g = 1080(1 - 0,325e^{(-0,167t)} - 0,675e^{-2,5t}) + 20. \qquad (6)$$

The convective heat transfer coefficient is: $\alpha_c = 50Wt/m^2$.

The calculation of reinforced concrete and steel-reinforced concrete structures consists of two stages.

The first stage begins with determining the load-bearing capacity at a normal temperature of 20 °C, i.e. using Eurocode 2 EN 1992–1-1:2010 or DBN V.2.6–98:2009. These standards propose using equations to describe the relationship between σ_c і ε_c for short-term axial loading. Equation (7), which is used in Eurocode 2 EN 1992–1–1:2010

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1+(k-2)\eta}, \tag{7}$$

where $\eta = \varepsilon_c / \varepsilon_{c1}$ – ratio of deformations in compressed concrete; ε_{c1} – deformations at maximum stresses, according to the table $k = \dfrac{1,05E_{cm}|\varepsilon_{c1}|}{f_{cm}}$ – coefficient according to DBN V.2.6-98:2009; E_{cm} – modulus of elasticity, according to Table 3.1; f_{cm} – maximum stresses, according to the table 3.1.

Equation (7) and equation (8) in the form of a fifth-degree polynomial are based on the results of numerous experimental studies conducted by the Scientific Research Institute of Building Structures, the statistical processing of which made it possible to propose a more complete regulatory framework.

$$\sigma_c = f_{(ck),(cd)} \sum_{k=1}^{5} a_k \eta^k. \tag{8}$$

Both formulas have defined limits of use

$$0 < \varepsilon_c | < |\varepsilon_{cu1}|, \tag{9}$$

where ε_{cu1} – nominal limit deformations of concrete.

Work [7] shows that the upward branches of the graphs according to DBN and EN 1992–1–1 practically coincide. Both normative documents provide concrete classes, values of relative concrete compression deformation ε_{c1} at maximum stresses f_{cm}, nominal concrete deformation limits ε_{cu1} (values of relative concrete compression deformation limits) and the average value of the initial concrete elastic modulus E_{cm} (GPa). The data given in Tables 3.1 and 3.2 are sufficient for calculating reinforced concrete structures at a temperature of 20 °C. In this case, the concrete strength classes, concrete compression deformation ε_{c1} at maximum stresses f_{cm}, nominal concrete deformation limits ε_{cu1}, average value of the initial modulus of elasticity of concrete E_{cm} average value of the initial modulus of elasticity of concrete.

The stress-strain relationship takes the form

$$\sigma_{c,\theta=20} = f_{cm,\theta=20} \frac{k\eta - \eta^2}{1+(k-2)\eta}, \tag{10}$$

where:

$$\varepsilon_c \div \varepsilon_{c1,\theta=20} = \varepsilon_c \div 0,0021;\ k_{\theta=20} = 1,05E_{cm,\theta=20}|\varepsilon_{c1,\theta=20}| \div f_{cm,\theta=20} =$$

$$= 1,05 \cdot 31000 \cdot 0,0021 \div 30 = 2,2785\ МПа;$$

$$\sigma_{cm,\theta=20} = 30\cdot\left(2278,5 \cdot \varepsilon_c / 0,0021 - (\varepsilon_c \div 0,0021)^2\right) \div (1 + (2,2785-2)/(\varepsilon_c \div 0,0021).$$

When calculating structures under high-temperature influences, it is necessary to take into account the temperature regime of the fire and its duration. The load-bearing capacity and deformability of building structures exposed to fire are influenced by the physical and mechanical properties of the construction material, which change depending on the heating temperature. In particular, these properties are determined by the strength limit and elastic modulus of the material from which the structures are made. The dependence of the change in strength characteristics was obtained using the least squares method for prismatic strength and has the following mathematical expression [8, 9].

The formulas obtained using the least squares method for the calculated strength value are 5th degree polynomials [10]:

– for expanded clay concrete in the range from 60 °C to 700 °C;

$$\alpha_\theta = \frac{f_{cd,\theta}}{f_{cd}} = 4,519 \cdot 10^{-30}\theta^5 - 1,299 \cdot 10^{-25}\theta^4 + 1,652 \cdot 10^{-22}\theta^3 - 2,555 \cdot 10^{-6}\theta^2 + \\ + 1,020 \cdot 10^3\theta + 0,98, \tag{11}$$

– for heavy concrete in the range from 60 °C to 700 °C;

$$\alpha_\theta = \frac{f_{cd,\theta}}{f_{cd}} = 4,303 \cdot 10^{-28}\theta^5 - 7,555 \cdot 10^{-25}\theta^4 + 4,863 \cdot 10^{-22}\theta^3 - 3,608 \cdot 10^{-6} \cdot \theta^2 + \\ + 2,320 \cdot 10^3\theta + 0,6184. \tag{12}$$

To assess the impact of high temperatures, the relationships given in design codes, in particular in standard EN 1992–1–2, which are generally accepted in modern structural design, are widely used in engineering practice. However, the use of code-specified values alone does not always provide the required level of accuracy, since the actual behaviour of materials may differ significantly from the standardised relationships due to various technological and structural factors. In this regard, it becomes particularly important to compare the theoretical curves obtained according to the standard with the results of full-scale tests, which makes it possible to identify general patterns of strength reduction, clarify the nature of changes in material properties, and objectively evaluate the adequacy of the adopted calculations.

Figure 3 presents comparative plots of the relative strength of lightweight expanded clay concrete and heavy concrete according to EN 1992-1-2 and the results of experimental studies.

Table 1 summarises the numerical values of strength changes for the specified materials in the range from 20 °C to 1200 °C, which makes it possible not only to perform a quantitative analysis of the differences between the calculated and experimental results but also to observe the discrepancies at different stages of temperature exposure.

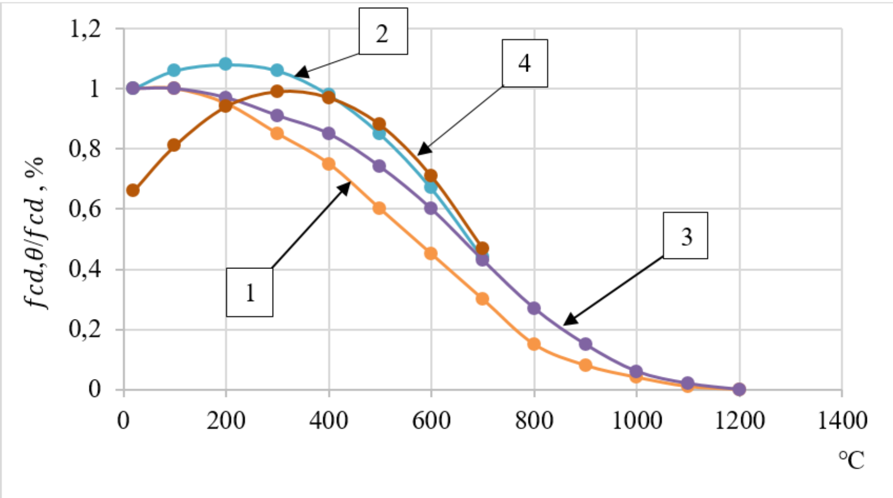

Fig. 3. Comparative graph showing the decrease in concrete strength with increasing temperature: 1) expanded clay concrete according to EN 1992-1-2; 2) expanded clay concrete according to the results of field tests; 3) heavy concrete according to EN 1992-1-2; 4) heavy concrete according to the results of field tests.

Table 1

Comparative table showing the decrease in concrete strength with increasing temperature

temperature of concrete °C	$\alpha_\theta = \frac{f_{cd,\theta}}{f_{cd}}$					
	Expanded clay concrete (silicate)		Difference	Heavy concrete (carbonate)		Difference
	EN 1992-1-2	Testing		EN 1992-1-2	Testing	
20	1,0	1,0	0,0	1,0	0,66	-0,34
100	1,0	1,06	0,06	1,0	0,81	-0,19
200	0,95	1,08	0,13	0,97	0,94	0,03
300	0,85	1,06	0,21	0,91	0,99	-0,08
400	0,75	0,98	0,22	0,85	0,97	-0,12
500	0,60	0,85	0,25	0,74	0,88	-0,14
600	0,45	0,67	0,22	0,60	0,71	-0,11
700	0,30	0,44	0,14	0,43	0,47	0,04
800	0,15	-	-	0,27	-	-
900	0,08	-	-	0,15	-	-
1000	0,04	-	-	0,06	-	-
1100	0,01	-	-	0,02	-	-
1200	0,00	-	-	0,00	-	-

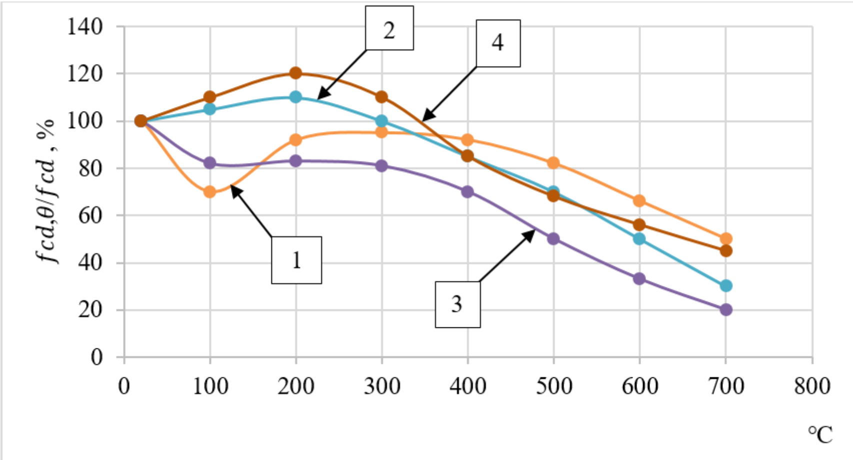

Fig. 4. The effect of high temperatures on the calculated compressive strength of concrete: 1) ordinary heavy concrete; 2) expanded clay concrete; 3) high-strength concrete; 4) expanded clay-perlite concrete

Table 2

Decrease in concrete compressive strength with increasing temperature

temperature (°C)	Regular heavy concrete	Expanded clay concrete	High-strength concrete	Expanded clay aggregate concrete
1	2	3	4	5
20	100	100	100	100
100	70	105	82	110
200	92	110	83	120
300	95	100	81	110
1	2	3	4	5
400	92	85	70	85
500	82	70	50	68
600	66	50	33	56
700	50	30	20	45

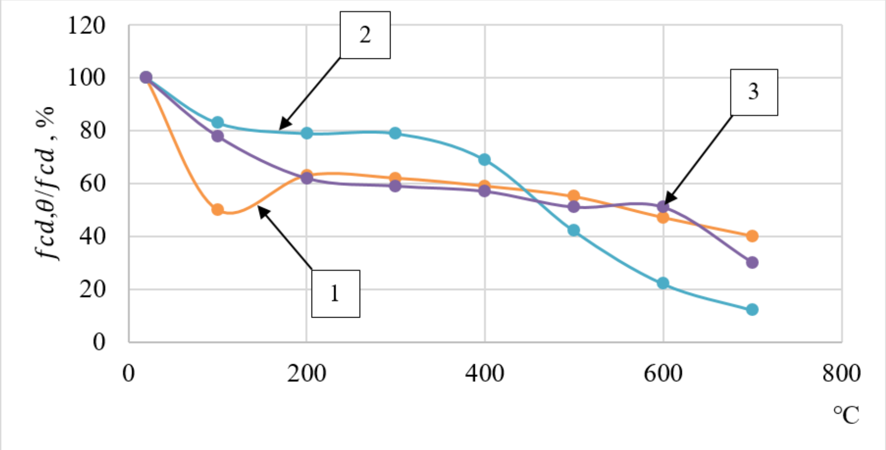

Fig. 5. The effect of high temperatures on the calculated tensile strength of concrete:
1) heavy concrete; 2) high-strength concrete; 3) expanded clay-perlite concrete

Table 3

Decrease in concrete tensile strength with increasing temperature

temperature (°C)	Heavy concrete	High-strength concrete	Expanded clay aggregate concrete
20	100	100	100
100	50	83	78
200	63	79	62
300	62	79	59
400	59	69	57
500	55	42	51
600	47	22	51
700	40	12	30

An increase in the heating temperature of the material contributes to a decrease in its initial elastic modulus [11]. A general view of the change in the elastic modulus of concrete with temperature is shown in Figure 6.

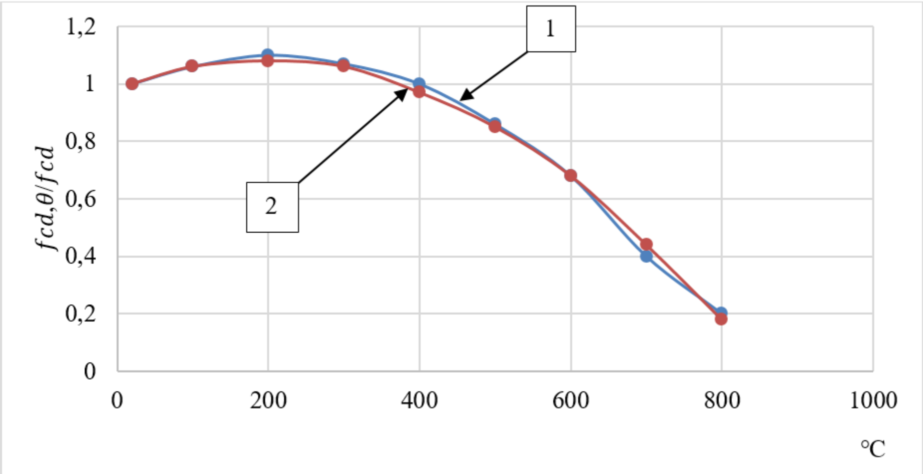

Fig. 6. Dependency graphs "$f_{cd,\theta} / f_{cd} - \theta$" for expanded clay concrete: 1) normative; 2) experimental

Table 4

The significance of the dependence of the initial modulus of elasticity $f_{cd,\theta} / f_{cd} - \theta$ for expanded clay concrete at elevated temperatures

temperature, °C	Normative	Experimental
20	1	1
100	1,06	1,06
200	1,1	1,08
300	1,07	1,06
400	1	0,97
500	0,86	0,85
600	0,68	0,68
700	0,4	0,44
800	0,2	0,18

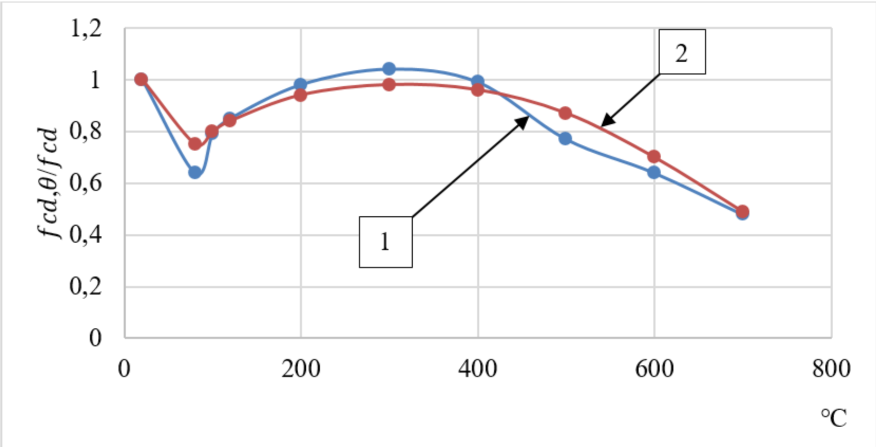

Fig. 7. Graphs showing the dependence of the initial modulus of elasticity $f_{cd,\theta} / f_{cd} - \theta$ on temperature for heavy concrete: 1) normative; 2) experimental

Table 5

The significance of the dependence of the initial modulus of elasticity $f_{cd,\theta} / f_{cd} - \theta$ for heavy concrete

temperature, °C	Normative	Experimental
20	1	1
80	0,64	0,75
100	0,79	0,8
120	0,85	0,84
200	0,98	0,94
300	1,04	0,98
400	0,99	0,96
500	0,77	0,87
600	0,64	0,7
700	0,48	0,49

The dependence of the initial modulus of elasticity of concrete on temperature can be obtained by analogy with the dependencies for concrete strength using the least squares method.

The formulas obtained using the least squares method for the initial modulus of elasticity are as follows [8, 9]:

– for expanded clay concrete in the range from 120 °C to 800 °C

$$\beta_\theta = \frac{E_{cm,\theta}}{E_{cm}} = 0,00367\left(\frac{\theta}{100} - 16,47\right)^2 + 0,022, \tag{12}$$

– for heavy concrete in the range from 120 °C to 800 °C

$$\beta_\theta = \frac{E_{cm,\theta}}{E_{cm}} = 1,2 - \frac{\theta}{100}\left(0,14 + 0,0012\frac{\theta}{100}\right). \tag{13}$$

The graphs of the dependence of the initial modulus of elasticity of concrete $E_{cm,\theta} / E_{cm} - \theta$ for expanded clay concrete and heavy concrete, which are constructed according to formulas (12) and (13), are shown in Figures 9 and 10, respectively. Curves 1 are constructed based on data from [7]; theoretical curves 2 are constructed according to formula (12) for expanded clay concrete and formula (13) for heavy concrete.

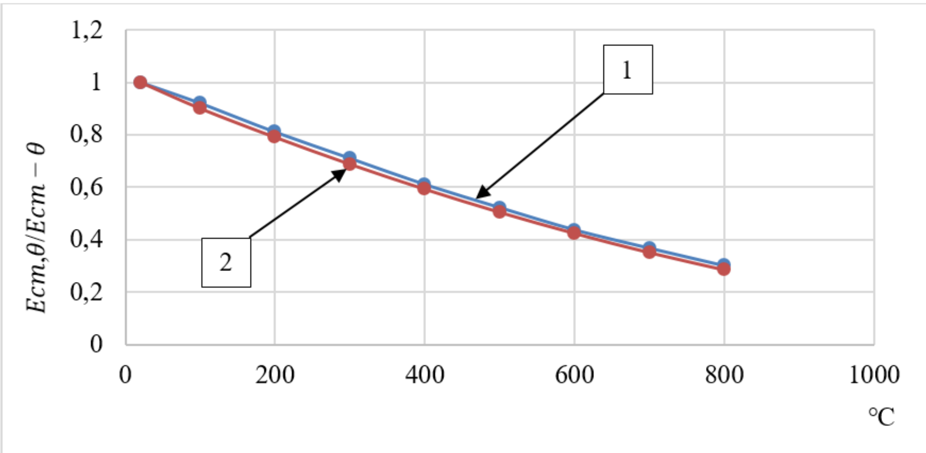

Fig. 8. Graphs showing the dependence of the initial modulus of elasticity Ecm, θ/Ecm - on temperature for expanded clay concrete: 1) according to data from A.F. Milovanov; 2) according to formula 12

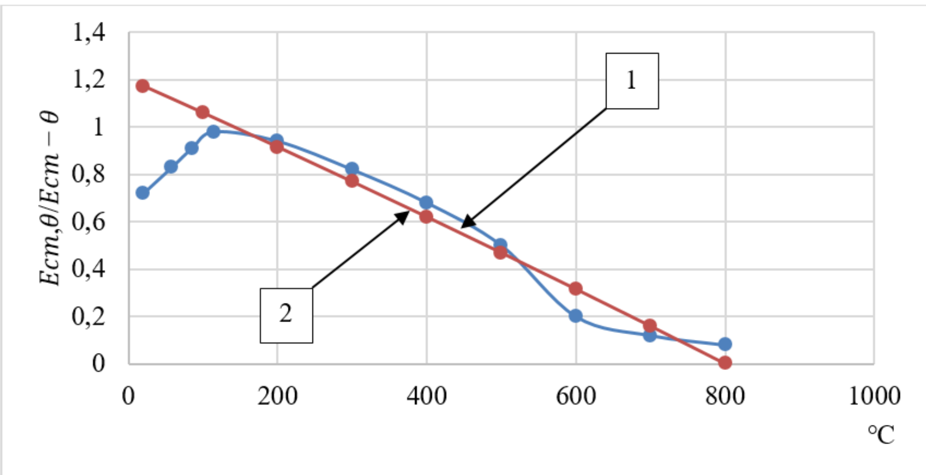

Fig. 9. Graphs showing the dependence of the initial modulus of elasticity Ecm, θ/Ecm- on temperature for heavy concrete: 1) according to data from A.F. Milovanov; 2) according to formula 13

The reduction in the calculated resistance of reinforcing steel can be taken into account by introducing a coefficient of reduction in the characteristic value of resistance depending on temperature $k_{y,\theta}$, the function for calculating which can be established in the first approximation based on the experimental data obtained [9].

For different classes of reinforcement, using the least squares method, the following analytical dependencies have been established, comparative graphs of which are shown in Figure 10:

– for reinforcement of class A240C:

$$k_{(y,\theta)}^{A240} = 1 - 0{,}00134\theta + 1{,}6 \cdot 10^{-7}\theta^2, \tag{14}$$

– for reinforcement of class A400C:

$$k_{(y,\theta)}^{A400} = 0{,}91 - 0{,}0004\theta - 1{,}06 \cdot 10^{-6}\theta^2, \tag{15}$$

– for reinforcement of class A500C:

$$k_{y,\theta}^{A500} = 0{,}942 - 0{,}00046\theta - 1{,}15 \cdot 10^{-6}\theta^2, \tag{16}$$

– for reinforcement of class A600C:

$$k_{y,\theta}^{A600} = 0{,}951 - 0{,}00025\theta - 1{,}15 \cdot 10^{-6}\theta^2. \tag{17}$$

In addition, an equation was obtained for determining the average value of the resistance reduction coefficient:

$$k_{y,\theta}^{red} = 1 - 0{,}0006\theta - 1{,}0 \cdot 10^{-6}\theta^2. \tag{18}$$

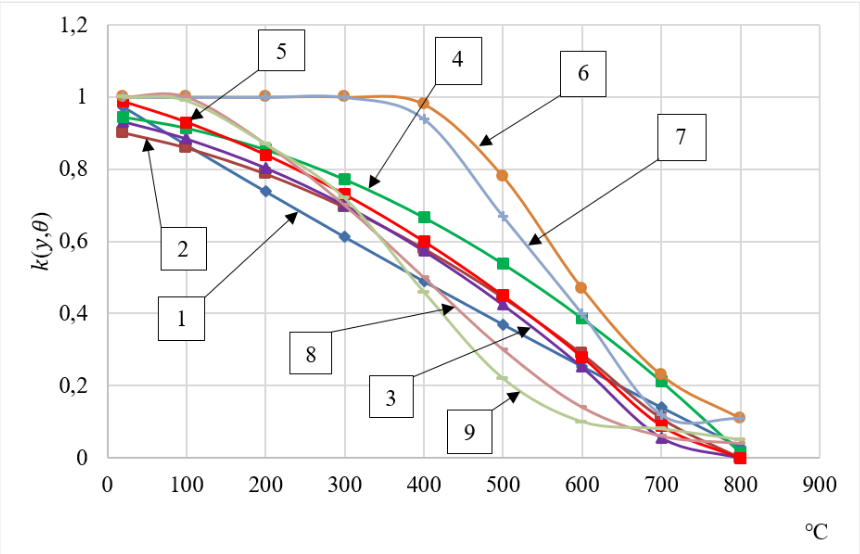

Fig. 10. Comparative graph of values of coefficients of reduction in the calculated resistance of reinforcement depending on temperature ky,θ: – according to experimental data [3]: 1) reinforcement class A240C; 2) reinforcement class A400C; 3) reinforcement class A500C; 4) class A600C reinforcement; 5) average value of the reduction coefficient; – according to EN 1992–1–1: 6) hot-rolled non-prestressed reinforcement; 7) cold-formed non-prestressed reinforcement; 8) class A pre-stressed reinforcement; 9) class B pre-stressed reinforcement

The strength and deformation properties of reinforcement at elevated temperatures are determined according to the stress-strain diagram EN 1992–1–2.

The thermal deformation of concrete $\varepsilon_{s_i}(\theta)$ is defined as the relative change in the length of a concrete sample when heated to a temperature θ, compared to its length at a temperature of 20 °C

$$\varepsilon_{s_i}(\theta)=\frac{l(\theta)-l_0}{l_0}, \tag{19}$$

where: $l(\theta)-$ length of the sample at temperature θ; l_0- length of the sample at 20 °C.

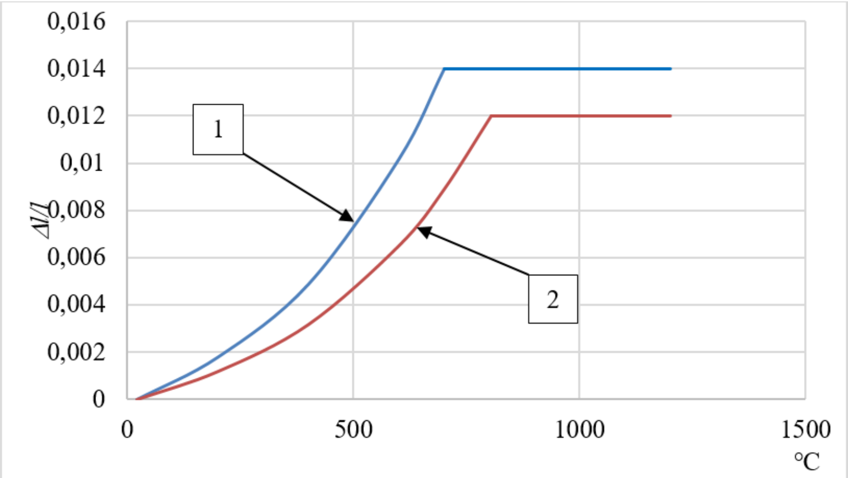

Fig. 11. Graph showing the dependence of thermal expansion on temperature: 1) silicate aggregate; 2) carbonate aggregate

The dynamic behaviour of reinforced concrete structures under the influence of high temperatures and explosive loads can be described by the dynamic equilibrium equation in the finite element method

$$M\ddot{u}(t)+C\dot{u}+K(T)u=F_{explosion}(t)+F_{therm}(T), \tag{20}$$

where $u-$ vector of nodal displacements, m; $\ddot{u}-$vector of accelerations, m/s^2; $\dot{u}-$ vector of velocities, m/s; $M-$mass matrix, kg; $C-$ damping matrix (Rayleigh model), N·s/m; $K(T)-$stiffness matrix taking into account temperature, N/m; $F_{explosion}(t)-$ impulse load from explosion, H; $F_{therm}(T)-$vector of thermal forces from thermal expansion, N.

Temperature-dependent stiffness matrix.

The temperature-dependent stiffness matrix of an element is determined by integration over its volume, $K_e(T)$

$$\mathrm{K_e}(T)=\int\Omega_e \mathrm{B(T)D}_{з/б}(T)\mathrm{B}\Omega, \tag{21}$$

where $D_{з/б}(T)-$effective elasticity matrix of reinforced concrete at the corresponding temperature; $B-$ deformation geometry matrix; $\Omega_в-$integration area

The effective elasticity matrix is defined as the sum of the contributions of concrete and reinforcement, $D_{з/б}(T)$.

$$D_{з/б}(T)=V_{concrete}D_{concrete}^{compression}(T)+V_{reinforcement}D_{reinforcement}^{stretch}(T)+V_{reinforcement}D_{reinforcement}^{compression}(T), \tag{22}$$

where: $V_{concrete}, V_{\text{reinf}\,orcement}$ – modulus of elasticity of concrete depending on temperature; $D_{concrete}^{compression}(T)$ – concrete matrix under compression at the corresponding temperature; $D_{reinforcement}^{stretch}(T)$ – reinforcement matrix under tension at the corresponding temperature; $D_{reinforcement}^{compression}(T)$ – reinforcement matrix under compression at the corresponding temperature.

Concrete stiffness matrix.

Concrete works effectively in compression, so its matrix is determined only for the region where ε <0 (i.e., negative deformations) and has the form

$$D_{concrete}^{compression}(T)=\begin{cases}\dfrac{E_{concrete}(T)}{1-\mu_{concrete}^{2}}\begin{bmatrix}1 & \mu_{concrete} & 0\\ \mu_{concrete} & 1 & 0\\ 0 & 0 & \dfrac{1-\mu_{concrete}}{2}\end{bmatrix}, at\,\varepsilon<0\\ 0, \quad at\,\varepsilon\geq 0\end{cases} \tag{23}$$

where $E_{concrete}(T)$ – modulus of elasticity of concrete depending on temperature; $\mu_{concrete}$ – Poisson's ratio for concrete.

Reinforcement stiffness matrix.

Reinforcement in reinforced concrete structures works effectively in both tension and compression. To describe its contribution to stiffness characteristics, it is advisable to distinguish between two cases:

– tensile deformations $\varepsilon>0$;

– compressive deformations $\varepsilon<0$.

In the finite element method, these cases are taken into account by introducing separate reinforcement stiffness matrices: tensile matrices and compressive matrices:

$$D_{reinf.}^{stretch}(T)=\begin{cases}\dfrac{E_{reinf.}(T)}{1-\mu_{reinf.}^{2}}\begin{bmatrix}1 & \mu_{reinf.} & 0\\ \mu_{reinf.} & 1 & 0\\ 0 & 0 & \dfrac{1-\mu_{reinf.}}{2}\end{bmatrix}, at\,\varepsilon>0\\ 0, \quad at\,\varepsilon\leq 0\end{cases} \tag{24}$$

$$D_{reinf.}^{compression}(T)=\begin{cases}\dfrac{E_{reinf.}(T)}{1-\mu_{reinf.}^{2}}\begin{bmatrix}1 & \mu_{reinf.} & 0\\ \mu_{reinf.} & 1 & 0\\ 0 & 0 & \dfrac{1-\mu_{reinf.}}{2}\end{bmatrix}, at\,\varepsilon<0\\ 0, \quad at\,\varepsilon\geq 0\end{cases} \tag{25}$$

where $E_{reinf.}(T)$ – modulus of reinforcement resistance depending on temperature; $\mu_{concrete}$ – Poisson's ratio for reinforcement; ε – deformation (the sign determines the mode: tension or compression).

Damping matrix.

The damping matrix $C(T)$, is defined as a linear combination of mass and stiffness matrices according to Rayleigh's model

$$C(T) = a_M M + \beta_K K(T), \tag{26}$$

where α_M – mass damping coefficient; β_K – stiffness damping coefficient; M – mass matrix, kg; $K(T)$ – temperature-dependent stiffness matrix of reinforced concrete.

Mass matrix.

The volumetric mass matrix M, which is used in dynamic finite element method problems, has the general form

$$M = \int \Omega \rho \; N^T N \, d\Omega, \tag{27}$$

where ρ – material density (constant for each element type), kg/m^3; Ω – element volume; N – matrix of shape functions (interpolation functions).

Impulse load from an explosion.

In the finite element method, an impulse load $F_{explosion}(t)$, is modelled as a temporary concentrated force or pressure applied to the surface of a structure (nodes) for a very short time.

The general expression for the force from an explosion is as follows

$$F_{explosion}(t) = P(t)S, \tag{28}$$

where: $P(t)$ – time-varying explosion pressure., Pa; S – area vector subjected to pressure, m^2.

The explosion pressure function P(t) can be described using simplified and more realistic models. A triangular impulse is used for approximate calculations, while an exponential model is used for more accurate reproduction of the time variation of the load.

The triangular impulse (the simplest option) looks as follows:

$$P(t) = P_{max.}\left(1 - \frac{t}{t_d}\right), 0 \le t \le t_d, \tag{29}$$

where $P_{max.}$ – maximum explosion pressure, Pa; t, t_d – duration of shock wave action, attenuation, ms.

A more realistic representation of the temporal change in pressure is provided by the exponential model

$$P(t) = P_{max.} \; e^{-t/\tau}, \tag{30}$$

where τ – explosion pressure decay time, ms.

Temperature deformations.

Under high temperatures that occur during a fire, structural materials undergo thermal expansion, which is accompanied by an increase in internal deformations and stresses. This additional load is reflected in the form of a vector of thermal forces $F_T(T)$.

In the finite element method, it is formed by integrating temperature deformations within the volume of the element:

$$F_{temp.}(T) = \int \Omega_{concrete} \mathrm{B}^T \mathrm{D}_{concrete} \; (\mathrm{T}) \; \varepsilon_{temp.}^{concrete} d\Omega + \int \Omega_{reinf.} \; \mathrm{B}^T \; \mathrm{D}_{reinf.}(T) \varepsilon_{temp.}^{reinf.} d, \tag{31}$$

where: B^T – matrix of geometric relationships; $D_{concrete}(T)$, $D_{reinfor.}(T)$ – stiffness matrices for concrete and reinforcement at respective temperatures; $\Omega_{concrete}$, $\Omega_{reinfor.}$ – areas of concrete and reinforcement, respectively; $\varepsilon_{temp.}^{concrete}$, $\varepsilon_{temp.}^{reinfor.}$ – temperature deformation of concrete and reinforcement.

The temperature deformation of the material $\varepsilon_{temp.}^{m.}$ determined by the dependence

$$\varepsilon_{temp.}^{concrete} = \alpha_m(T)\Delta T \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{32}$$

where $\alpha_m(T)-$ coefficient of linear thermal expansion of material, *m*; $\Delta T - T - T_0 -$ temperature change, *K*.

A comparison of the code-specified relationships given in EN 1992-1-2 with the results of full-scale experimental studies revealed significant discrepancies, particularly in the reduction of the strength of heavy concrete and lightweight expanded clay aggregate concrete, as well as in the variation of the initial modulus of elasticity with temperature. This highlights the need to refine analytical models to ensure that calculations more accurately reflect actual operating conditions.

The analytical relationships obtained for the strength, modulus of elasticity, and thermal deformation of concrete, as well as reduction factors for reinforcement strength at elevated temperatures, provide a comprehensive basis for predicting the behaviour of structures in fire conditions.

The use of the proposed refined mathematical relationships improves the accuracy of calculations when assessing the robustness of monolithic reinforced concrete buildings against progressive collapse caused by fire and deflagration explosions.

The mathematical model, implemented within the finite element method framework, accounts for temperature effects, impulsive explosive loads, and the behaviour of concrete and reinforcement at elevated temperatures. This enables an adequate representation of the dynamic behaviour of reinforced concrete structures under fire and explosion conditions. At the same time, the model remains simplified and does not account for several important physical factors, in particular crack formation in concrete, creep of materials at high temperatures, and the spatial effects of blast wave propagation.

5 DISCUSSION OF THE RESULTS OF THE STUDY

The results of the study demonstrate that the improved mathematical model offers significant advancements in predicting the fire resistance of reinforced concrete structures. By incorporating transient heat transfer, thermal expansion, and the effects of impulsive explosive loads, the model ensures a more realistic assessment of the dynamic behaviour of structures compared to conventional approaches. This holistic integration of physical processes contributes to the reliability of performance predictions under extreme conditions.

A key finding is the discrepancy identified between experimental data and the simplified code-specified curves for both concrete and reinforcement at elevated temperatures. Such inconsistencies highlight the limitations of existing standards and confirm the necessity of revising current design practices. The developed polynomial relationships and analytical expressions for the reduction factors of strength and elasticity provide more precise tools for engineers, allowing for improved prediction accuracy when assessing structural performance under fire exposure.

The successful implementation of the refined thermomechanical model within the finite element method framework further emphasizes its practical relevance. The model not only enhances the accuracy of evaluating load-bearing capacity and deformability but also creates opportunities for its application in methodologies aimed at assessing structural robustness and preventing progressive collapse. In particular, its capacity to capture the combined effects of fire and explosion represents a novel contribution to safety analysis, broadening the scope of available engineering tools.

Overall, the findings underline the importance of moving towards analytical models that better reflect real material behaviour under high-temperature and dynamic loading scenarios. The study thus provides both theoretical contributions to the development of advanced models and practical implications for improving the fire and explosion resilience of monolithic reinforced concrete structures.

6 CONCLUSIONS

1. The components of an improved mathematical model for the fire resistance of reinforced concrete structures have been defined. The model takes into account transient heat transfer, thermal expansion of materials, and the effect of impulsive explosive loads, which enables an adequate reproduction of the dynamic behaviour of structures. A comparison of the characteristics of concrete and reinforcement at elevated temperatures with code-specified curves and the results of full-scale tests confirmed the existence of significant discrepancies that reduce the accuracy of calculations.
2. Based on the analysis of experimental data, polynomial relationships were developed to describe the reduction in strength and initial modulus of elasticity of heavy concrete and lightweight expanded clay aggregate concrete, and analytical expressions were obtained for reduction factors for the strength of reinforcement of different grades at elevated temperatures. This allows simplified code-specified curves to be replaced with more accurate analytical relationships, thereby improving the accuracy of predicting the behaviour of materials when exposed to high fire temperatures.

An improved thermomechanical model has been implemented within the finite element method framework for assessing the load-bearing capacity and deformability of structures under the combined effects of fire and explosion. The model can be used as a tool in methodologies for evaluating the robustness of monolithic reinforced concrete buildings against progressive collapse caused by fire and explosion.

7 ETHICAL DECLARATIONS

The authors of the article have no relevant financial or non-financial interests to disclose.

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For references:
R. Maiboroda, Y. Otrosh. (2025). Improved thermomechanical model for predicting the behaviour of reinforced concrete structures under fire and explosion conditions**.** Mechanics and Mathematical Methods. VII (2). 66–85.

Для посилань:
Майборода Р. І., Отрош Ю. А. Удосконалена термомеханічна модель для прогнозування поведінки залізобетонних конструкцій в умовах пожежі та вибуху. Механіка та математичні методи, 2025. Т. VII. №2. С. 66–85.