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PERTURBED MOTIONS OF A NEARLY DYNAMICALLY SPHERICAL RIGID BODY WITH A MOVABLE MASS SUBJECT TO CONSTANT BODY-FIXED TORQUE

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Abstract: The problem of motion of a rigid body about a fixed point is one of the classical problems of mechanics. The interest to the problems of the rigid body dynamics has increased in the second half of the XX century in connection with the development of rocket and space technologies. A spacecraft or satellite, while orbiting about its center of mass, experiences torques from forces of diverse physical nature. This includes torques generated by the motion of internal masses, which can arise from factors such as presence of rotating components (like rotors or gyroscopes), and the activities of crew members aboard the crew vehicle. The dynamics of rigid body incorporated moving masses is a significant focal point in classical mechanics. Extensive research is dedicated to investigating the rotation of a rigid body featuring motion of internal masses. It is assumed that the body contains a viscoelastic element that is modeled by a moving mass connected to the body by a strong damper. The moving mass model loosely attached elements in a space vehicles, which can significantly affect the vehicle's motion about its center of mass during a long period of time. Some cases are considered of the motion of a rigid body containing internal masses connected to the body by means of elastic and dissipative elements. A number of works are devoted to the analysis of various problems of the dynamics of space vehicles containing internal movable masses.

The paper develops an approximate solution by means of averaging method to the system of Euler's equation terms for a nearly dynamically spherical rigid body containing a viscoelastic element under the action of constant body-fixed torque. We obtained the system of motion equations in the standard form which refined in square-approximation by small parameter. Asymptotic approach permits to obtain some qualitative results and to describe evolution of angular motion using simplified averaged equations and numerical solution. The main objective of this paper is to extend the previous results for the problem of motion about a center of mass of a rigid body under the influence of small internal torque (cavity filled with a fluid of high viscosity) or external torque (resistive medium). The importance of the results is in progress of moving mass control motion of spinning projectiles.

Keywords: nearly dynamically spherical rigid body, moving mass, constant torque.

ЗБУРЕНІ РУХИ БЛИЗЬКОГО ДО ДИНАМІЧНО СФЕРИЧНОГО ТВЕРДОГО ТІЛА З РУХОМОЮ МАСОЮ ПІД ДІЄЮ ПОСТІЙНИХ МОМЕНТІВ В ЗВ'ЯЗАНИХ З ТІЛОМ ОСЯХ

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Анотація: Проблема руху твердого тіла відносно нерухомої точки є однією з класичних задач механіки. Інтерес до задач динаміки твердого тіла посилився в другій половині XX сторіччя в зв'язку з розвитком ракетно-космічної техніки.

Космічний корабель або супутник в своєму русі відносно центра мас зазнає вплив моментів сил різної фізичної природи. Це, наприклад, моменти, викликані рухом внутрішніх мас, які можуть виникати через такі фактори, як наявність обертових компонентів (роторів, гіроскопів), а також переміщенням екіпажу в випадку пілотованого апарату. Динаміка твердого тіла з рухомими масами це важлива проблема класичної механіки.

Велика кількість робіт присвячена дослідженню обертання твердого тіла з рухомими масами, з пружними та дисипативними елементами. Припускається що тіло містить в'язкопружний елемент, який моделюється рухомою масою з'єднаною демпфером з корпусом. Наявність рухомої маси моделює присутність нежорстко закріплених елементів на космічному апараті, що при тривалому періоді часу має суттєвий вплив на його рух відносно центра мас. Ряд публікацій присвячено аналізу різних проблем динаміки космічних апаратів, що містять внутрішні рухомі маси. Вивчались проблеми стійкості та нестійкості, а також проблеми керування і стабілізації рухів.

В статті проведене асимптотичне розв'язування за допомогою методу усереднення системи рівнянь Ейлера з додатковими збурюючими моментами для близького до динамічно сферичного твердого тіла з в'язкопружним елементом під дією сталого моменту в зв'язаних з тілом осях. Одержано систему рівнянь руху в стандартній формі, яка уточнена в квадратичному наближенні за малим параметром.

За допомогою асимптотичного підходу одержані якісні результати та описана еволюція руху тіла з допомогою усереднених рівнянь і чисельного інтегрування. В роботі розвинуті результати досліджень попередніх задач, розглянутих авторами, про рух твердого тіла під дією моментів, обумовлених порожниною з рідиною великої в'язкості або середовища з опором. Одержані результати важливі в процесі управління рухом тіла з масою або для рухів обертових снарядів з масою.

Ключові слова: близьке до динамічно сферичного тверде тіло, рухома маса, постійний момент.

1 INTRODUCTION

The analysis of objects containing elements with distributed and lumped parameters is of interest both from the theoretical points of view. Constructive results for systems containing quasi-rigid bodies have been obtained. These models assume that the motion is close to the motion of perfectly rigid bodies. The influence of nonidealities can be taken into account using asymptotic methods of nonlinear mechanics. It is reduced to including additional disturbing torques in the Euler equations of the angular motion of a fictitious rigid body. The dynamics of the motion of rigid bodies with internal degrees of freedom were studied in number of publications.

2 ANALYSIS OF LITERATURY DATA AND RESOLVING THE PROBLEM

The dynamics of a rigid body incorporating moving masses is a significant focal point in classical mechanics. Extensive research is dedicated to investigating the rotation of a rigid body featuring motion of internal masses. A number of problems in the indicated field and the works in this direction are described in [1-8].

In [1, 9], scenarios involving the motion of a rigid body containing movement of internal masses are explored. Several problems concerning the motion of a rigid body incorporating elastic and dissipative components are investigated in [10-13]. [14] tackled the issue of minimum-time deceleration in a resistant medium for the rotation of a dynamically symmetric rigid body containing a viscous-elastic element. [15] focused on the challenge of achieving quasi-optimal time-based deceleration for a gyrostat featuring a moving mass in a medium with resistance.

In [16] the influence is estimated of the moving point masses (linear oscillations) on the stability of uniform rotation of the Lagrange top.

Paper [17] delved into the motion of a rigid body that is close to dynamically spherical, and houses a cavity filled with a highly viscous fluid. In [18], researchers explored the motion of a nearly dynamically spherical rigid body, also with a cavity containing viscous fluid but at a low Reynolds member. They provided insights into both the qualitative and quantitative aspects of its motion in a resistive medium. [19] focused on the motion about the center of mass of a nearly dynamically spherical rigid body with a cavity filled with highly viscous fluid, which was subjected to constant body-fixed torque. The motion of a nearly dynamically spherical rigid body with highly viscous fluid under the action of constant body-fixed torques is investigated in [20].

In [21] qualitative and quantitative results of motion of a nearly dynamically spherical rigid body with a moving mass attached to the body by means of elastic coupling were presented. Paper [22] extended the investigation of rigid body motion presented in [17] by adding another (third) component of the gyrostatic moment. In [23] the results of [17] was generalized to charged rigid body.

The paper [24] study the motion about the center of mass of a nonsymmetric rigid body influenced by two small perturbation torques: a constant one in the body-fixed axes and a linear dissipative one depending on the angular velocity.

In the works [25, 26] analytical solutions are obtained for the problem of a rigid body by a torque which is constant in the body-fixed axes.

In paper [27], the analytic solution has been introduced for the rotation of a rigid body having spherical ellipsoid of inertia subjected to a constant torque.

Let us examine the motion of a dynamically asymmetric rigid body about its center of inertia, featuring a movable point mass m connected via an elastic linkage to a point O_1 located on one of its principal axes of inertia. We assume that the torques which is constant in the body-connected axes have the form

$$M_i^c = \varepsilon^2 M_i = \text{const},\ i = 1,2,3 \tag{1}$$

where $0 < \varepsilon << 1$ is a small parameter.

The origin of Cartesian coordinate system, connected with the rigid body, is placed at the center of inertia of the body with point mass, whereas the basic vectors e_1, e_2, e_3 of the system are directed so that vector e_3 coincides with axes on which point O_1 is located. Then radius-vector of point O_1, $\rho = \rho e_3$ where, we assume $\rho > 0$.

In references [1, 9], a vector equation was derived to describe the alteration of the vector $\boldsymbol{\omega}$ within the coordinate system linked to the body.

$$J_0^* \cdot \omega + \left(\omega \cdot J_0^* \cdot \omega\right) = \Phi(\omega) + O(\Omega^{-4}, \lambda^2\Omega^{-8}) \tag{2}$$

Here J_0^* denotes the inertia tensor of a rigid body containing a moving mass when referenced with respect to point O, ω is the absolute angular velocity of the body, vector function Φ includes the terms of the orders of Ω^{-2} and $\lambda\Omega^{-4}$. The quantities $\Omega^2 = c/m,\ \lambda = \delta/m$, characterize the frequency and decay time of free oscillations, c is a stiffness coefficient, and δ is a viscous friction coefficient.

Perturbation torques in (2) are small, provided

$$\Omega^2 \ll \lambda\omega \ll \omega^2 \tag{3}$$

Free oscillations of the system decaying long time before the body performs one revolution [1, 9].

To obtain equations (2) and assess their level of error, you can refer to [1, 9]. Function $\Phi(\omega)$ is a polynomial containing the fourth and fifth order of ω [1, 9].

Having evaluated the vector function Φ as per [1, 9], the equation (2) for our problem, when expressed in terms of projections on axes e_1, e_2, e_3, takes the shape

$$\begin{aligned}
&A\frac{dp}{dt} + (C-B)qr = -\rho^2 m\left\{\Omega^{-2} qr(Q_1 p^2 + K_1 q^2 + L_1 r^2) + \right.\\
&\qquad + \lambda\Omega^{-4} p[q^2(M_1 p^2 + N_1 q^2 + R_1 r^2) + r^2(S_1 p^2 + T_1 r^2)] + \varepsilon^2 M_1\\
&B\frac{dq}{dt} + (A-C)pr = -\rho^2 m\left\{\Omega^{-2} pr(Q_2 q^2 + K_2 p^2 + L_2 r^2) + \right.\\
&\qquad + \lambda\Omega^{-4} q[r^2(M_2 q^2 + N_2 r^2 + R_2 p^2) + p^2(S_2 q^2 + T_2 p^2)] + \varepsilon^2 M_2\\
&C\frac{dr}{dt} + (B-A)pq = -\rho^2 m\lambda\Omega^{-4} r^3(A+C-B)(B+C-A)\times\\
&\qquad \times A^{-1}B^{-1}[(A-C)B^{-1}p^2 + (B-C)A^{-1}q^2] + \varepsilon^2 M_3
\end{aligned} \tag{4}$$

The tensor of inertia J_0^* of the rigid body, whose point mass m is superposed with O_1, obtains the form

$$J_0^* = \begin{vmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{vmatrix}, \tag{5}$$

where A, B and C are the principal moments of inertia of the rigid body, p, q, r are the components of the absolute angular velocity ω.

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The right-hand sides of two equations (2) comprises coefficients $Q_1, K_1, L_1, M_1, N_1, R_1, S_1, T_1, Q_2, K_2, L_2, M_2, N_2, R_2, S_2, T_2$ that are specific expressions containing A, B, C. For instance,

$$Q_1 = -1 - 5\frac{A-C}{B} - 6\frac{C-B}{A} - 3\frac{B-A}{C} + 7\frac{(A-C)(B-A)}{BC} + 4\frac{(B-A)(C-B)(A-C)}{ABC}. \quad (6)$$

We have chosen not to reference in our paper other expressions from (4) such as $K_1, ..., T_2$ due to their drawbacks.

If $A = B$, $\varepsilon = 0$ the system (4) reduces to the corresponding system in [1, 9].

3 PURPOSE AND TASKS OF THE STADY

Take into consideration a nearly dynamically spherical rigid body, where the principal central moments of inertia of the unperturbed body can be expressed in the following manner

$$A = J_0 + \varepsilon A', \quad B = J_0 + \varepsilon B', \quad C = J_0, \quad (7)$$

where $0 < \varepsilon << 1$ is a small parameter.

According to (2) Ω^{-2}, $\lambda\Omega^{-4}$ are small parameters in equations of motion (4). We assume that, i.e. $\Omega^{-2} \sim \varepsilon^2$, $\lambda\Omega^{-4} \sim \varepsilon^2$, $\lambda\Omega^{-2} \sim 1$.

For $\varepsilon = 0$ equations of motion (4) depict the motion of a body exhibiting spherical symmetry.

We also make an assumption

$$|A - B| = O(\varepsilon^2 J_*), \quad |A' - B'| = O(\varepsilon J_*), \quad J_* \sim J_0. \quad (8)$$

Then, after (7), (8) the following equations are provided

$$A - B = \varepsilon(A' - B') = \varepsilon^2 J_*, \quad A - C = \varepsilon A', \quad B - C = \varepsilon B'. \quad (9)$$

After applying transformations to the system (4), we derive the perturbed Euler system. Relations (7)–(9) and transfer to show time $\tau = \varepsilon t$ are considered (terms of order ε^3 and higher are rejected):

$$\begin{aligned}
\frac{dp}{d\tau} &= \frac{B'}{J_0}\left(1 - \varepsilon\frac{A'}{J_0}\right)qr + \varepsilon f_{1p}(p,q,r), \qquad p(0) = p_0, \\
\frac{dq}{d\tau} &= -\frac{A'}{J_0}\left(1 - \varepsilon\frac{B'}{J_0}\right)pr + \varepsilon f_{1q}(p,q,r), \qquad q(0) = q_0, \\
\frac{dr}{d\tau} &= \frac{A' - B'}{J_0}pq + \varepsilon f_{1r}(p,q,r), \qquad r(0) = r_0.
\end{aligned} \quad (10)$$

In the given equation, r is a slow variable. The set of differential equations in (10) constitutes a nonlinear system, wherein the frequency is contingent upon the slow variable. The perturbations were incorporated within (10).

$$\varepsilon f_{1p}(p,q,r)=\frac{-\rho^2 m}{J_0}\left\{\Omega^{-1}qrh_{11}+\frac{\Omega^{-2}}{J_0}qr(h_{12}-A'h_{11})+\frac{\lambda\Omega^{-4}}{J_0}p(q^2h_{21}+r^2h_{31})\right\}+$$
$$+\varepsilon\frac{M_1}{J_0}(1-\varepsilon\frac{A'}{J_0}),$$
$$\varepsilon f_{1q}(p,q,r)=\frac{-\rho^2 m}{J_0}\left\{\Omega^{-1}prg_{11}-\frac{\Omega^{-2}}{J_0}pr(g_{12}+B'g_{11})+\frac{\lambda\Omega^{-4}}{J_0}q(r^2g_{21}+p^2g_{31})\right\}+ \quad (11)$$
$$+\varepsilon\frac{M_2}{J_0}(1-\varepsilon\frac{B'}{J_0}),$$
$$\varepsilon f_{1r}(p,q,r)=\frac{-\rho^2 m}{J_0^2}\lambda\Omega^{-4}r^3(A'p^2+B'q^2)+\varepsilon\frac{M_3}{J_0},$$
$$g_{11}=q^2+p^2+r^2,\qquad g_{12}=A'(p^2+3r^2)+(3A'-2B')q^2,$$
$$g_{21}=2(A'-B')p^2-B'r^2,\quad g_{31}=(A'-B')(q^2+p^2),$$
$$h_{11}=-g_{11},\qquad h_{12}=B'(q^2+3r^2)-(2A'-3B')p^2,$$
$$h_{21}=(B'-A')(q^2+p^2+2r^2),\qquad h_{31}=-A'r^2.$$

The perturbation torque of the influence of a movable mass in the rigid body is small [1, 9].

The solution of the unperturbed system (10) for $\varepsilon=0$ is as follows

$$p=a\cos\varphi,\quad q=-\frac{J_0 aw\sin\varphi}{B'r},\quad r=r_0. \qquad (12)$$

In the given equation, $a=\sqrt{p_0^2+(\dot p_0/w)^2}$ is the amplitude, $\varphi=w\tau+\varphi_0$ is the phase, $w=r\sqrt{A'B'}\big/J_0$, $A'B'>0$, φ_0 is the initial phase, $\cos\varphi_0=p_0/a$, $\sin\varphi_0=-q_0\sqrt{B'/A'}\;\big/a$ supposedly.

We transition from the slow variables p, q, r to the standard slow variables a, r and the phase φ by implementing a change in variables:

$$p=a\cos\varphi,\quad q=-\frac{J_0 aw\sin\varphi}{B'r},\quad r=r. \qquad (13)$$

We differentiate equations (13) considering a perturbed system. Through a series of transformations, we arrive at the system in its standard form, where the point represents the time derivative of τ)

$$\dot a\cos\varphi-a\dot\varphi\sin\varphi=-a\,w(r)\sin\varphi+\varepsilon f_{2p},$$
$$\dot a\sin\varphi+a\dot\varphi\cos\varphi=a\,w(r)\cos\varphi-\sqrt{\frac{B'}{A'}}\varepsilon f_{2q}, \qquad (14)$$
$$\dot r=\frac{B'-A'}{J_0}a^2\sqrt{\frac{A'}{B'}}\sin\varphi\cos\varphi+\varepsilon f_{2r},\quad w(r)=\frac{r}{J_0}\sqrt{A'B'},$$
$$\varepsilon f_{2p}=\varepsilon\frac{aA'}{J_0}w(r)\sin\varphi+\varepsilon f_{1p},\quad \varepsilon f_{2r}=\varepsilon f_{1r},\quad \sqrt{\frac{B'}{A'}}\,\varepsilon f_{2q}=\varepsilon\frac{aB'}{J_0}w(r)\cos\varphi+\sqrt{\frac{B'}{A'}}\varepsilon f_{1q}.$$

We will address the equations (14) with a focus on solving for $\dot a$ and $\dot\varphi$, resulting in the derivation of a system:

$$\dot{a} = \varepsilon f_{2p}\cos\varphi - \varepsilon f_{2q}\sqrt{\frac{B'}{A'}}\sin\varphi,$$
$$\dot{\varphi} = w(r) - \frac{1}{a}\varepsilon f_{2p}\sin\varphi - \frac{1}{a}\varepsilon f_{2q}\sqrt{\frac{B'}{A'}}\cos\varphi, \quad (15)$$
$$\dot{r} = \frac{B'-A'}{J_0}a^2\sqrt{\frac{A'}{B'}}\sin\varphi\cos\varphi + \varepsilon f_{2r}.$$

We insert (13) into the third equation (11) for the variable r. With the standard transformations, we arrive at the system of equations:

$$\dot{a} = \varepsilon\frac{A'-B'}{J_0}aw\sin\varphi\cos\varphi + \varepsilon f_{1p}\cos\varphi - \varepsilon f_{1q}\sqrt{\frac{B'}{A'}}\sin\varphi,$$
$$\dot{r} = \frac{B'-A'}{J_0}a^2\sqrt{\frac{A'}{B'}}\sin\varphi\cos\varphi - \quad (16)$$
$$-\frac{\rho^2 m}{J_0^2}\lambda\Omega^{-4}r^3\left(A'a^2\cos^2\varphi + B'\left(-\frac{J_0 aw}{B'r}\right)^2\sin^2\varphi\right) + \varepsilon\frac{M_3}{J_0},$$

$$\varepsilon f_{1p}\cos\varphi = \frac{-\rho^2 m}{J_0}\cos\varphi\left\{-\frac{J_0 aw\sin\varphi}{B'}\left[\left(\Omega^{-1} - \frac{\Omega^{-2}}{J_0}A'\right)\left(-r^2 - a^2\cos^2\varphi - \left(\frac{J_0 aw}{B'r}\right)^2\sin^2\varphi\right)\right] +\right.$$
$$+\frac{\Omega^{-2}}{J_0}\left[B'\left(3r^2 + \left(\frac{J_0 aw}{B'r}\right)^2\sin^2\varphi\right) - (2A'-3B')a^2\cos^2\varphi\right] + \frac{\lambda\Omega^{-4}}{J_0}a\cos\varphi\times$$
$$\left.\times\left[\left(\frac{J_0 aw}{B'r}\right)^2\sin^2\varphi\left(-2r^2 - a^2\cos^2\varphi - \left(\frac{J_0 aw}{B'r}\right)^2\sin^2\varphi\right) + A'r^4\right]\right\} + \varepsilon\frac{M_1}{J_0}\left(1-\varepsilon\frac{A'}{J_0}\right)\cos\varphi,$$

$$\varepsilon f_{1q}\sin\varphi = \frac{-\rho^2 m}{J_0}\sin\varphi\left\{ra\cos\varphi\left[\left(\Omega^{-1} - \frac{\Omega^{-2}}{J_0}B'\right)\left(r^2 + a^2\cos^2\varphi + \left(\frac{J_0 aw}{B'r}\right)^2\sin^2\varphi\right) -\right.\right.$$
$$-\frac{\Omega^{-2}}{J_0}\left(A'(3r^2 + a^2\cos^2\varphi) + (3A'-2B')\left(\frac{J_0 aw}{B'r}\right)^2\sin^2\varphi\right)\Big] + \frac{\lambda\Omega^{-4}}{J_0}\left(-\frac{J_0 aw}{B'r}\sin\varphi\right)\times$$
$$\left.\times\left[r^2\left(2a^2\cos^2\varphi(A'-B') - B'r^2\right) + a^2\cos^2\varphi(A'-B')\left(a^2\cos^2\varphi + \left(\frac{J_0 aw}{B'r}\right)^2\sin^2\varphi\right)\right]\right\} +$$
$$+\varepsilon\frac{M_2}{J_0}\left(1-\varepsilon\frac{B'}{J_0}\right)\sin\varphi.$$

In this system of equations, the value $w(r) = r\sqrt{A'B'}/J_0$ represents the perturbed frequency of the converted system. If we average system (16) over the phase φ [28], we obtain:

$$\dot{a} = \beta\gamma(\alpha a^5 + \eta a^3 r^2 - ar^4),$$
$$\dot{r} = \beta A'a^2 r^3 + \varepsilon\frac{M_3}{J_0}. \quad (17)$$

The notations are introduced at this point:

$$\beta = \frac{-\rho^2 m}{J_0^2}\lambda\Omega^{-4}, \quad \eta = \frac{1}{2}\left(1-\frac{A'}{B'}\right), \quad \alpha = \frac{1}{8}\left(1-\frac{A'^2}{B'^2}\right), \quad \gamma = \frac{1}{2}(A'-B').$$

We transform system (17) to:

$$\begin{aligned}\frac{dx}{d\tau} &= 2\beta\gamma x(\alpha x^2 + \eta xy - y^2),\\ \frac{dy}{d\tau} &= 2\beta A' xy^2 + \varepsilon \frac{2M_3}{J_0}\sqrt{y}.\end{aligned} \tag{18}$$

Here we include the slow variables $x = a^2$, $y = r^2 > 0$ in system (18).

3 BASIC RESULTS

System (18) was numerically resolved using the initial conditions $x(0) = 1$, $y(0) = 1$ and task factors $m = 1$, $\rho = 1$, $\varepsilon = 0.1$, constant moment projection $M_3 = -0.135$. The values of the components of the moments of inertia are presented in Table 1.

The graphical representations of the varying values $x = a^2$ and $y = r^2$ (the squared equatorial and axial components of the rigid body angular velocity vector) are represented in two cases (Fig. 1-4) when parameters are $\lambda = 98$, $\Omega = 10$ and $\lambda = 9$, $\Omega = 3$.

Table 1

Components of moments of inertia.

Case	J_0	A'	B'
1	1	0,51	0,5
2	3	0,83	0,8

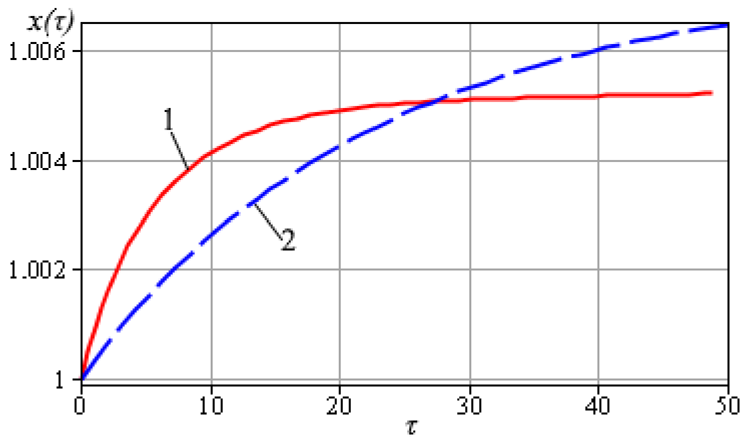

Fig. 1. The plots of changing value x in the cases (1) and (2) for $\lambda = 98$, $\Omega = 10$

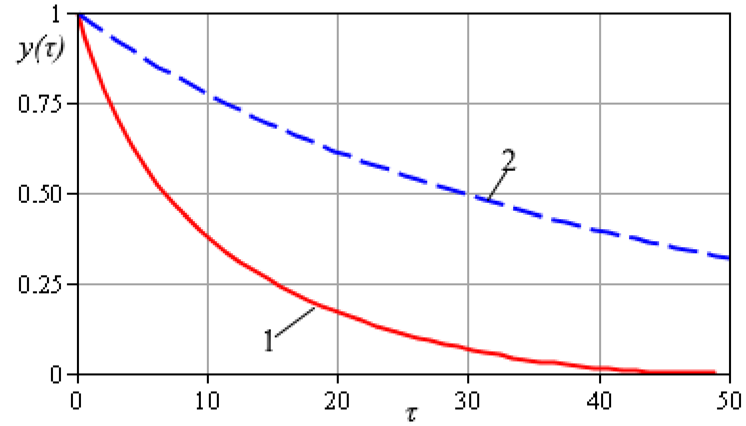

Fig. 2. The plots of changing value y in the cases (1) and (2) for $\lambda = 98, \quad \Omega = 10$

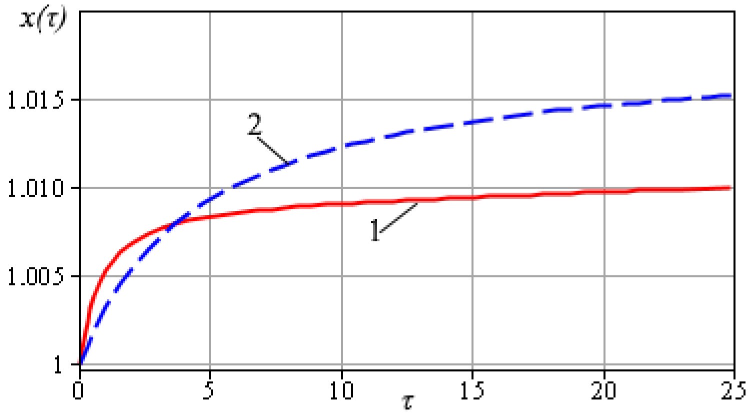

Fig. 3. The plots of changing value x in the cases (1) and (2) for $\lambda = 9, \quad \Omega = 3$

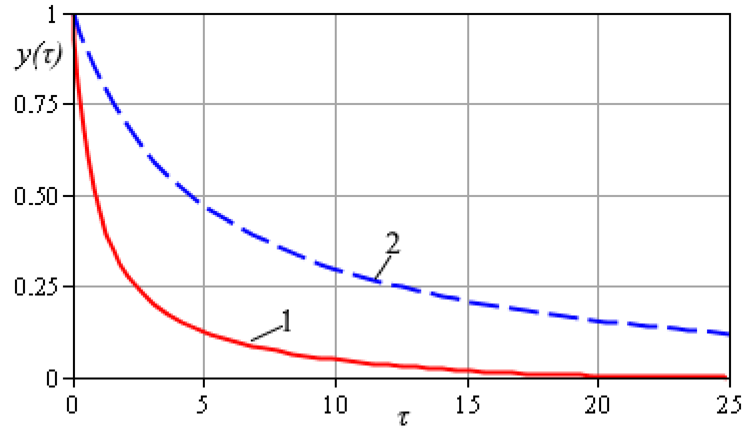

Fig. 4. The plots of changing value y in the cases (1) and (2) for $\lambda = 9, \quad \Omega = 3$

4 DISCUSSION OF THE RESULTS OF THE STUDY

Variable x (Fig. 1, 3) has a slight increase, maximum value which is achieved in case 2 ($A/C < 1.5$) at $\lambda = 9$, $\Omega = 3$ and is equal to $x = 0.016$. However, in case 1 ($A/C \sim 1.5$), the growth rate is observed at the initial values of time, regardless of the values λ, Ω, satisfying the condition (3).

The variable $y = r^2$ (Figs. 2, 4) asymptotically approaches zero. In the case of a ratio of moments of inertia $A/C \sim 1.5$ (the first case), the decrease y occurs faster than in the case of $A/C < 1.5$ (the second case). Also, the nature of the decrease of the axial component depends on the values of λ, Ω. When $\lambda = 9$, $\Omega = 3$ the axial component decreases faster than when $\lambda = 98$, $\Omega = 10$ (with the other parameters being the same).

5 CONCLUSIONS

The motion of a nearly dynamically spherical rigid body with a movable mass under the action of constant body-fixed torques is investigated. We obtain the system of motion equations in standard form, which refined in square approximation by small parameter.

The averaging method was applied to the nonlinear system of rotational motion equations. The evolution of rigid body motion is described. The importance of the results is to applications such as analyzing angular motions of spacecraft, in moving mass control, and reentry vehicles.

6 ETHICAL DECLARATIONS

The authors have no relevant financial or non-financial interests to report.

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