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FORCED VIBRATIONS OF ARCH SYSTEMS IN ITS PLANE

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Abstract: The work is devoted to solving the problem of forced oscillations of a circular arch using the numerical-analytical method of boundary elements. The algorithm of the method can conditionally be divided into two parts - analytical and numerical. The first of this two parts involves obtaining in an analytical form a complete system of fundamental solutions of the original differential equation, constructing the Green's function and the components of the vector of external loads, which are the problems to be solved in this article. An ordinary sixth-order differential equation is obtained that describes the forced oscillations of an arch in its plane. It differs from the similar equation for free oscillations obtained earlier only in the presence of the right-hand side. This means that, as with free oscillations, 10 solutions are possible here, and the analytical expressions derived from 360 fundamental functions for these solutions remain unchanged. For one of the variants of the roots of the characteristic equation, an analytical expression of the Green's function is constructed, a connection is established between the Green's function and one of the fundamental functions, which is also valid for other values of the roots of the characteristic equation. Using impulse functions and splines, the arch load vector is constructed.

The presented work implements the analytical component of the numerical-analytical method of boundary elements. The numerical implementation of the algorithm and the comparison of the results with the results of finite element analysis determine the direction of further research.

It is noted that the cost of computer resources when implementing a program for calculating an arch system using the boundary element method is minimal, since it is necessary to solve a system of only twelve algebraic equations, which is significantly less than when using the finite element method. The results obtained allow us to perform dynamic calculations for forced vibrations any arched systems of arbitrary configuration.

Keywords: boundary elements method, circular arch, forced vibrations, fundamental functions, Green's function.

ВЫНУЖДЕННЫЕ КОЛЕБАНИЯ АРОЧНЫХ СИСТЕМ В СВОЕЙ ПЛОСКОСТИ

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Аннотация: Работа посвящена решению задачи о вынужденных колебаниях круговой арки числено-аналитическим методом граничных элементов. Алгоритм метода условно можно разбить на две части – аналитическую и численную. Первая из них предполагает получение в аналитическом виде полной системы фундаментальных решений исходного дифференциального уравнения, построение функции Грина и компонентов вектора внешних нагрузок, что и является задачами, решаемыми в данной статье. Получено обыкновенное дифференциальное уравнение шестого порядка, описывающее вынужденные колебания арки в своей плоскости. Оно отличается от аналогичного уравнения при свободных колебаниях, полученного ранее, только наличием правой части. Это означает, что, как и при свободных колебаниях, здесь возможны 10 вариантов решения, причем, остаются неизменными выведенные ранее аналитические выражения 360 фундаментальных функций для этих решений. Для одного из вариантов корней характеристического уравнения построено аналитическое выражение функции Грина, установлена связь между функцией Грина и одной из фундаментальных функций, которая справедлива и для других значений корней характеристического уравнения. С использованием импульсных функций и сплайнов построен вектор нагрузки на арку.

Представленная работа реализует аналитическую составляющую численно-аналитического метода граничных элементов. Численная реализация алгоритма и сравнение полученных результатов с результатами конечно-элементного анализа определяют направление дальнейших исследований.

Отмечено, что затраты компьютерных ресурсов при реализации программы расчета арочной системы методом граничных элементов являются минимальными, так как приходится решать систему только двенадцати алгебраических уравнений, что существенно меньше, чем при использовании метода конечных элементов.

Ключевые слова: метод граничных элементов, круговая арка, вынужденные колебания, фундаментальная функция, функция Грина.

1 INTRODUCTION

Arches are perhaps the most ancient building structures. As supporting structures they were used as far back as Assyria. The first steel arches appeared in the 40s of the XIXth century. Subsequently, they became widespread in various fields. Currently, arches are used for industrial and public buildings, hangars, warehouses, greenhouses, etc. In construction, arched bridges of various designs are widespread. Their calculation for static loads has been developed quite fully, and for dynamic loads нагрузки much worse. Meanwhile, a significant increase in vehicle speeds and an increase in their carrying capacity leads to an increase in static and dynamic loads. In addition, new high-strength materials have appeared, structural forms and calculation methods are being improved, and all this leads to a decrease in the material consumption of structures, and, consequently, to their greater sensitivity to dynamic effects due to a decrease in rigidity.

In this regard, the development of methods for solving problems of free and forced vibrations of arches and arch systems continues to be relevant.

2 ANALYSIS OF LITERATURE DATA AND PROBLEM FORMULATION

The first publications devoted to the dynamic calculations of arches and arch systems appeared a long time ago, and were primarily associated with the construction of bridges. These are the works of F. Kh. Brown [1], K. Federhofer [2], A. B. Morgaevsky [3], I. M. Rabinovich [4], N. K. Snitko [5], A. F. Smirnov [6] and several other researchers. Then, over a sufficiently long period, no serious results were observed in the field of the dynamics of arches. And with the development of computer technology and the advent of new methods of calculating work in this direction, especially in foreign countries, they have intensified.

P. Chidamparam and A. W. Leissa [7] summarized all the literature published at that time (1993) on the vibrations of curved rods, beams, rings, and arches of arbitrary shape. The extensive bibliography cited in their article includes 407 references, including to the few works of Russian-speaking authors. However, the vast majority of work was related to the use of numerical methods and, mainly, the finite element method and its various modifications. There are a lot of such works, we note only some of them [8-13], which seem to be the most interesting. The trend towards the use of numerical methods continued in the future. There are also publications in the form of articles [14, 15] or sections in books [16, 18], where analytical approaches are proposed, however, there are many limitations associated with loads, geometry, or boundary conditions.

3 AIM AND PURPOSE OF RESEARCH

The aim of this work is to solve the problem of forced vibrations of the arch by the numerical-analytical method of boundary elements (NA BEM), the main provisions of which are described in [19-21].

The application of the proposed approach has already made it possible to obtain solutions to a wide class of problems of statics, dynamics and stability of rod systems, plates and shells, including a number of problems for which such solutions were absent, and also proved to be very effective for systems with discrete-continuous distribution of parameters .

The algorithm of the NA BEM can conditionally be divided into two parts - analytical and numerical. The first of these involves obtaining in an analytical form a complete system of fundamental solutions of the original differential equation, constructing the Green's function and the components of the vector of external loads, which are the problems to be solved in this article.

4 RESEARCH RESULTS

Let us consider the forced oscillations of a circular arch occurring under the action of normal $q_n(\alpha,t)$ (Fig. 1) and tangential $q_t(\alpha,t)$ (Fig. 2) forcing loads of arbitrary form.

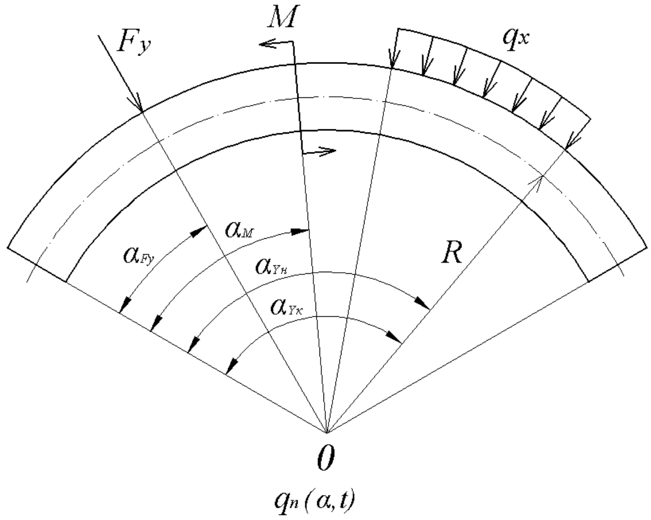

Fig. 1. Circular arch under normal driving forces

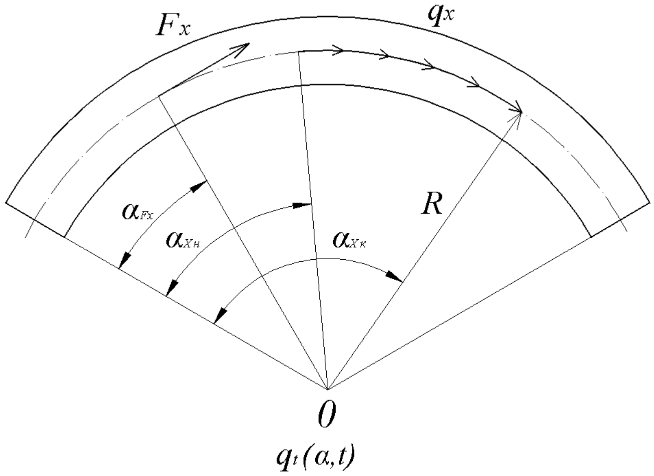

Fig. 2. Circular arch, tangential driving forces

In this case, the equilibrium equations [5] will be supplemented by the corresponding terms:

$$\frac{\partial Q(\alpha,t)}{\partial \alpha} = -N(\alpha,t) + mR\frac{\partial^2 v(\alpha,t)}{\partial t^2} - q_n(\alpha,t)R; \tag{1}$$

$$\frac{\partial N(\alpha,t)}{\partial \alpha} = -Q(\alpha,t) + mR\frac{\partial^2 u(\alpha,t)}{\partial t^2} - q_t(\alpha,t)R. \tag{2}$$

And if we take into account the inertia forces, the presence of which is due to angular displacements, then the third equilibrium equation has the form

$$\frac{\partial M(\alpha,t)}{\partial \alpha} = Q(\alpha,t)R - \rho I\left[\frac{\partial^2 u(\alpha,t)}{\partial t^2} + \frac{\partial^3 v(\alpha,t)}{\partial \phi \partial t^2}\right], \tag{3}$$

where ρ – material density; I – axial moment of inertia of the cross section; m – linear mass.

The second term in the right-hand side of (3) represents the moment of inertia of rotation

$$M_{ин}(\alpha,t) = -\rho I \varepsilon ds = -\rho I\left[\frac{\partial^2 u(\alpha,t)}{\partial t}+\frac{\partial^2 v(\alpha,t)}{\partial\varphi\partial t^2}\right],$$

where

$\varepsilon = \partial^2\phi(\alpha,t)/\partial t^2$ – angular acceleration of the angle of rotation of the cross section of the arch.

Taking into account the inertia of rotation significantly complicates the third equation of equilibrium, but does not significantly affect the final result, therefore, in the future we will use (3) in a simplified form

$$\frac{\partial M(\alpha,t)}{\partial\alpha} = Q(\alpha,t)R. \tag{4}$$

Using the known relations between stresses, deformations and displacements, we express the normal force and bending moment through displacements u and v:

$$\begin{aligned} N &= EA\varepsilon = \frac{EA}{R}(u'-v); \\ M &= -EI\chi = -\frac{EI}{R^2}(u'+v''). \end{aligned} \tag{5}$$

In view of (4) and (5), equations (1) and (2) are written as:

$$-\frac{EI}{R^3}\frac{\partial^3 u(\alpha,t)}{\partial\alpha^3}-\frac{EI}{R^3}\frac{\partial^4 v(\alpha,t)}{\partial\alpha^4}+\frac{EA}{R}\frac{\partial u(\alpha,t)}{\partial\alpha}-\frac{EA}{R}v(\alpha,t)-mR\frac{\partial^2 v(\alpha,t)}{\partial t^2}=-q_n(\alpha,t)R;$$

$$\frac{EA}{R}\frac{\partial}{\partial\alpha}\left[\frac{\partial u(\alpha,t)}{\partial\alpha}-v(\alpha,t)\right]+\frac{EI}{R^3}\frac{\partial}{\partial\alpha}\left[\frac{\partial u(\alpha,t)}{\partial\alpha}+\frac{\partial^2 v(\alpha,t)}{\partial\alpha^2}\right]-mR\frac{\partial^2 u(\alpha,t)}{\partial t^2}=-q_t(\alpha,t)R.$$

We apply the Fourier method of variables separation to these equations, setting

$$u(\alpha,t)=u(\alpha)T(t),\ \ q_n(\alpha,t)=q_n(\alpha)T(t),$$
$$v(\alpha,t)=v(\alpha)T(t),\ \ q_t(\alpha,t)=q_t(\alpha)T(t),$$

then

$$-\frac{EI}{R^3}u'''T-\frac{EI}{R^3}v^{IV}T+\frac{EA}{R}u'T-\frac{EA}{R}vT-mRvT''=-q_n(\alpha)TR,$$
$$\frac{EA}{R}u''T-\frac{EA}{R}v'T+\frac{EI}{R^3}(u''+v''')T-mRuT''=-q_t(\alpha)TR.$$

Now deriving from frequency equation $T''+\omega^2T=0$, we substitute T'' with $(-\omega^2T)$. After substitution and reduction by T we will obtain

$$\begin{aligned} &v^{IV}+\frac{EAR^2-m\omega^2R^4}{EI}v+u'''-\frac{EAR^2}{EI}u'=\frac{R^4}{EI}q_n(\alpha), \\ &\left(1+\frac{EAR^2}{EI}\right)u''+\frac{m\omega^2R^4}{EI}u+v'''-\frac{EAR^2}{EI}v'=-\frac{R^4}{EI}q_t(\alpha). \end{aligned} \tag{6}$$

Assuming $\varepsilon=0$, that is, neglecting the tensile strain, we have

$$u'(\alpha) = v(\alpha). \tag{7}$$

Taking into account (7), we differentiate on α the first of equations (6) and add it to the second

$$u^{VI} + 2u^{IV} + \left(1 - \frac{m\omega^2 R^4}{EI}\right)u'' + \frac{m\omega^2 R^4}{EI}u = \frac{R^4}{EI}(q_n' - q_t). \tag{8}$$

As expected, the obtained equation of forced oscillations of the arch in its plane (8) differs from the analogous equation for free oscillations only in the presence of the right-hand side. This means that, as with free oscillations, 10 solutions are possible here, and the 360 fundamental functions obtained by us earlier for these solutions remain unchanged.

In accordance with the algorithm of the CA MGE [21], it is now necessary to form a vector of the right parts - a vector of loads, which, in turn, involves the calculation of the Green's function and its first five derivatives.

Let us find out what fundamental functions are associated with the Green function and its derivatives. For this, we consider one of the solutions to the differential equation of free oscillations [19], in which the corresponding characteristic equation has roots

$$t_1 < 0,\ t_{2,3} = \alpha \pm i\beta.$$

The general solution is written as

$$u(\alpha) = C_1\Phi_1(\alpha) + C_2\Phi_2(\alpha) + C_3\Phi_3(\alpha) + C_4\Phi_4(\alpha) + C_5\Phi_5(\alpha) + C_6\Phi_6(\alpha).$$

Functions $\Phi_1 - \Phi_6$ take values

$$\begin{aligned} &\Phi_1 = \cos\sqrt{t_1}\alpha, \quad \Phi_2 = \sin\sqrt{t_1}\alpha, \quad \Phi_3 = ch\gamma\alpha\sin\delta\alpha, \quad \Phi_4 = ch\gamma\alpha\cos\delta\alpha, \\ &\Phi_5 = sh\gamma\alpha\cos\delta\alpha, \quad \Phi_6 = sh\gamma\alpha\sin\delta\alpha. \end{aligned} \tag{9}$$

The algorithm for constructing the Green's function is independent of the boundary conditions of the problem.

Let us define the integration constants $C_k(\xi)$ $(k = 1, 2, ..., 6)$ from linear system of equations at $\alpha = \xi$:

$$\begin{vmatrix} y_1(\xi) & y_2(\xi) & y_3(\xi) & y_4(\xi) & y_5(\xi) & y_6(\xi) \\ y_1'(\xi) & y_2'(\xi) & y_3'(\xi) & y_4'(\xi) & y_5'(\xi) & y_6'(\xi) \\ y_1''(\xi) & y_2''(\xi) & y_3''(\xi) & y_4''(\xi) & y_5''(\xi) & y_6''(\xi) \\ y_1'''(\xi) & y_2'''(\xi) & y_3'''(\xi) & y_4'''(\xi) & y_5'''(\xi) & y_6'''(\xi) \\ y_1^{1V}(\xi) & y_2^{1V}(\xi) & y_3^{1V}(\xi) & y_4^{1V}(\xi) & y_5^{1V}(\xi) & y_6^{1V}(\xi) \\ y_1^{V}(\xi) & y_2^{V}(\xi) & y_3^{V}(\xi) & y_4^{V}(\xi) & y_5^{V}(\xi) & y_6^{V}(\xi) \end{vmatrix} \begin{vmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1/a_0 \end{vmatrix}. \tag{10}$$

The determinant of system (10) is Vronsky's determinant, which is not equal to zero for $\alpha \in (0, \phi_{гр})$, therefore, system (10) has a unique solution.

Let us form partial solution:

$$Y(\alpha,\xi) = C_1(\xi)y_1(\alpha) + C_2(\xi)y_2(\alpha) + C_3(\xi)y_3(\alpha) + C_4(\xi)y_4(\alpha) + C_5(\xi)y_5(\alpha) +$$

$$+ C_6(\xi)y_6(\alpha).$$

We write the Green's function in the form

$$G(\alpha,\xi) = Y(\alpha,\xi)H(\alpha - \xi), \tag{11}$$

where $H(\alpha-\xi)$ – Heaviside function [22].

If we stipulate in advance that the Green inequality $\alpha>\xi$ always holds, then the Heaviside function in (11) can be omitted.

Provided that the coefficient at the highest degree in the main differential equation is equal to the unity, we obtain the system of linear algebraic equations

$$\begin{cases} y_1C_1+y_2C_2+y_3C_3+y_4C_4+y_5C_5+y_6C_6=0; \\ y_1'C_1+y_2'C_2+y_3'C_3+y_4'C_4+y_5'C_5+y_6'C_6=0; \\ y_1''C_1+y_2''C_2+y_3''C_3+y_4''C_4+y_5''C_5+y_6''C_6=0; \\ y_1'''C_1+y_2'''C_2+y_3'''C_3+y_4'''C_4+y_5'''C_5+y_6'''C_6=0; \\ y_1^{1V}C_1+y_2^{1V}C_2+y_3^{1V}C_3+y_4^{1V}C_4+y_5^{1V}C_5+y_6^{1V}C_6=0; \\ y_1^{V}C_1+y_2^{V}C_2+y_3^{V}C_3+y_4^{V}C_4+y_5^{V}C_5+y_6^{V}C_6=1, \end{cases} \tag{12}$$

where

$$y_1=\cos\sqrt{t_1}\alpha;\ \ y_2=\sin\sqrt{t_1}\alpha;\ \ y_3=ch\gamma\alpha\sin\delta\alpha;$$
$$y_4=ch\gamma\alpha\cos\delta\alpha;\ \ y_5=ch\gamma\alpha\cos\delta\alpha;\ \ y_6=ch\gamma\alpha\sin\delta\alpha.$$

Solving system (12) by the Gauss method $(\alpha=\xi)$, we obtain:

$$\begin{cases} C_1=-\dfrac{\sin\sqrt{t_1}\xi}{a\sqrt{t_1}}; \\ C_2=\dfrac{\cos\sqrt{t_1}\xi}{a\sqrt{t_1}}; \\ C_3=\dfrac{\gamma(t+\gamma^2-3\delta^2)ch\gamma\xi\cos\delta\xi+\delta(t+3\gamma^2-\delta^2)sh\gamma\xi\sin\delta\xi}{2\gamma\delta a(\gamma^2+\delta^2)}; \\ C_4=\dfrac{-\gamma(t+\gamma^2-3\delta^2)ch\gamma\xi\sin\delta\xi+\delta(t+3\gamma^2-\delta^2)sh\gamma\xi\cos\delta\xi}{2\gamma\delta a(\gamma^2+\delta^2)}; \\ C_5=\dfrac{\gamma(t+\gamma^2-3\delta^2)sh\gamma\xi\sin\delta\xi-\delta(t+3\gamma^2-\delta^2)ch\gamma\xi\cos\delta\xi}{2\gamma\delta a(\gamma^2+\delta^2)}; \\ C_6=\dfrac{-\gamma(t+\gamma^2-3\delta^2)sh\gamma\xi\cos\delta\xi-\delta(t+3\gamma^2-\delta^2)ch\gamma\xi\sin\delta\xi}{2\gamma\delta a(\gamma^2+\delta^2)}, \end{cases}$$

where

$$a=t^2+2t(\gamma^2-\delta^2)+(\gamma^2+\delta^2)^2.$$

Thus, the Green’s function after substitution and simple transformations takes the form

$$G(\alpha,\xi)=C_1(\xi)y_1(\alpha)+C_2(\xi)y_2(\alpha)+C_3(\xi)y_3(\alpha)+C_4(\xi)y_4(\alpha)+C_5(\xi)y_5(\alpha)+$$
$$+C_6(\xi)y_6(\alpha)=\frac{\sin\sqrt{t_1}(\alpha-\xi)}{a\sqrt{t_1}}+\frac{1}{2\gamma\delta a(\gamma^2+\delta^2)}[\gamma(t+\gamma^2-3\delta^2)ch\gamma(\alpha-\xi)\sin\delta(\alpha-\xi)- \tag{13}$$
$$-\delta(t+3\gamma^2-\delta^2)sh\gamma(\alpha-\xi)\cos\delta(\alpha-\xi)].$$

It is easy to verify that function (13) has all the properties characteristic of the Green function:

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$- G(\alpha,\xi)=0$ at $\alpha<\xi$.

$- G(\alpha,\xi)$, as function of α at constant ξ in $(0,\phi_{гр})$, with exception of point $\alpha=\xi$, satisfies to linear homogenous differential equation.

$- G(\alpha,\xi)$ and its derivatives by α up to n-th order inclusive are continuous for $\alpha\in(0,\phi_{гр})$ excluding point $\alpha=\xi$, in which derivatives by α are continuous only up to $(n-2)$ order, and $(n-1)$ derivative has a gap of the first kind with a jump

$$\left.\frac{d^{(n-1)}G(\alpha,\xi)}{d\alpha^{(n-1)}}\right|_{x=\xi+0}-\left.\frac{d^{(n-1)}G(\alpha,\xi)}{d\alpha^{(n-1)}}\right|_{x=\xi-0}=\frac{1}{a_0}.$$

− At $\alpha=\xi$

$$G(\xi,\xi)=G'(\xi,\xi)=...=G^{(n-2)}(\xi,\xi)=0,\ \ G^{(n-1)}(\xi,\xi)=\frac{1}{a_0}.$$

$- G(\alpha,\xi)$ for equations with permanent coefficients depends only on difference of two variables $(\alpha-\xi)$

Using the notation (9), the Green's function (13) can be represented as

$$G(\alpha,\xi)=\frac{\Phi_2(\alpha-\xi)}{a\sqrt{t_1}}+\frac{(t+\gamma^2-3\delta^2)\Phi_3(\alpha-\xi)}{2\delta a(\gamma^2-\delta^2)}-\frac{(t+3\gamma^2-\delta^2)\Phi_5(\alpha-\xi)}{2\gamma a(\gamma^2+\delta^2)}. \tag{14}$$

A comparison of (14) with the expressions of fundamental functions obtained earlier [19] shows that

$$G(\alpha,\xi)=-A_{56}(\alpha-\xi). \tag{15}$$

The first five derivatives of the Green's function are expressed as follows:

$$G'(\alpha,\xi)=\frac{\Phi_1(\alpha-\xi)}{a}-\frac{\Phi_4(\alpha-\xi)}{a}+\frac{(t+\gamma^2-\delta^2)\Phi_6(\alpha-\xi)}{2\gamma\delta a}$$

or

$$G'(\alpha,\xi)=-A_{16}(\alpha-\xi). \tag{16}$$

$$G''(\alpha,\xi)=\frac{\sqrt{t_1}\Phi_2(\alpha-\xi)}{a}+\frac{(t+\gamma^2+\delta^2)}{2\delta a}+\frac{(t-\gamma^2-\delta^2)\Phi_5(\alpha-\xi)}{2\gamma a}$$

or

$$G''(\alpha,\xi)=A_{14}(\alpha-\xi). \tag{17}$$

$$G'''(\alpha,\xi)=\frac{t_1\Phi_1(\alpha-\xi)}{a}+\frac{t_1\Phi_4(\alpha-\xi)}{a}+\frac{[t_1(\gamma^2-\delta^2)+(\gamma^2+\delta^2)^2]\Phi_6(\alpha-\xi)}{2\gamma\delta a}$$

or

$$G'''(\alpha,\xi)=A_{24}(\alpha-\xi). \tag{18}$$

$$G^{IV}(\alpha,\xi)=\frac{t_1\sqrt{t_1}\Phi_2(\alpha-\xi)}{a}+\frac{[t_1(\gamma^2-3\delta^2)+(\gamma^2+\delta^2)^2]\Phi_3(\alpha-\xi)}{2\delta a}+$$

$$+\frac{[t_1(3\gamma^2-\delta^2)+(\gamma^2+\delta^2)^2]\Phi_5(\alpha-\xi)}{2\gamma a}$$

or

$$G^{IV}(\alpha,\xi) = A_{34}(\alpha-\xi)\,. \tag{19}$$

$$G^{V}(\alpha,\xi) = \frac{t_1^2 \Phi_1(\alpha-\xi)}{a} + \frac{[2t_1(\gamma^2-\delta^2)+(\gamma^2+\delta^2)^2]\Phi_4(\alpha-\xi)}{a} +$$
$$+\frac{[t_1(\gamma^4-6\gamma^2\delta^2+\delta^4)+(\gamma^2-\delta^2)(\gamma^2+\delta^2)^2]\Phi_6(\alpha-\xi)}{2\gamma\delta a}$$

or

$$G^{V}(\alpha,\xi) = A_{44}(\alpha-\xi)\,. \tag{20}$$

Let's move on to determining the components of the external load vector

$$\vec{B}^T = \left| B_{11} \quad B_{21} \quad B_{31} \quad B_{41} \quad B_{51} \quad B_{61} \right|. \tag{21}$$

$$B_{11} = \int_0^\alpha G(\alpha-\xi)q(\xi)d\xi\,, \tag{22}$$

where according to (15),

$$G(\alpha-\xi) = -A_{56}(\alpha-\xi)\,.$$

The right side of the differential equation of forced oscillations (8) in the load vector is taken into account in the form

$$q(\xi) = \frac{R^4}{EI}[q_n'(\xi) - q_t(\xi)]. \tag{23}$$

The components of this expression (Fig. 1, 2) are determined by the formulas:

$$q_t(\xi) = \frac{F_x}{R}\delta(\xi-\alpha_{F_x}) + q_x[H(\xi-\alpha_{x_H}) - H(\xi-\alpha_{x_K})]; \tag{24}$$

$$q_n(\xi) = \frac{F_y}{R}\delta'(\xi-\alpha_{F_y}) + \frac{M}{R^2}\delta'(\xi-\alpha_M) + q_y[H(\xi-\alpha_{y_H}) - H(\xi-\alpha_{y_K})]; \tag{25}$$

$$q_n'(\xi) = \frac{F_y}{R}\delta'(\xi-\alpha_{F_y}) + \frac{M}{R^2}\delta''(\xi-\alpha_M) + q_y[\delta(\xi-\alpha_{y_H}) - \delta(\xi-\alpha_{y_K})]. \tag{26}$$

Substituting (24) and (26) into (23), we obtain:

$$B_{11} = -\frac{R^4}{EI}\int_0^\alpha A_{56}(\alpha-\xi)\left\{\frac{F_y}{R}\delta'(\xi-\alpha_{F_y}) + \frac{M}{R^2}\delta''(\xi-\alpha_M) + q_y[\delta(\xi-\alpha_{y_H}) -\right.$$
$$\left.-\delta(\xi-\alpha_{y_K})] - \frac{F_x}{R}\delta(\xi-\alpha_{F_x}) - q_x[H(\xi-\alpha_{x_H}) - H(\xi-\alpha_{x_K})]\right\}d\xi =$$

$$= -\frac{F_y R^3}{EI}\int_0^\alpha A_{56}(\alpha-\xi)\delta'(\xi-\alpha_{F_y})d\xi - \frac{MR^2}{EI}\int_0^\alpha A_{56}(\alpha-\xi)\delta''(\xi-\alpha_M)d\xi -$$
$$-\frac{q_y R^4}{EI}\int_0^\alpha A_{56}(\alpha-\xi)\delta(\xi-\alpha_{y_H})d\xi + \frac{q_y R^4}{EI}\int_0^\alpha A_{56}(\alpha-\xi)\delta(\xi-\alpha_{y_K})d\xi +$$
$$+\frac{F_x R^3}{EI}\int_0^\alpha A_{56}(\alpha-\xi)\delta(\xi-\alpha_{F_x})d\xi + \frac{q_x R^4}{EI}\int_0^\alpha A_{56}(\alpha-\xi)H(\xi-\alpha_{x_H})d\xi -$$

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$$-\frac{q_xR^4}{EI}\int_0^{\alpha}A_{56}(\alpha-\xi)H(\xi-\alpha_{x_K})d\xi=-\frac{F_yR^3}{EI}A'_{56}(\alpha-\alpha_{F_y})+\frac{MR^2}{EI}A''_{56}(\alpha-\alpha_M)-$$
$$-\frac{q_yR^4}{EI}A_{56}(\alpha-\alpha_{y_H})+\frac{q_yR^4}{EI}A_{56}(\alpha-\alpha_{y_K})+\frac{F_xR^3}{EI}A_{56}(\alpha-\alpha_{F_x})+$$
$$+\frac{q_xR^4}{EI}\int_{\alpha_{x_H}}^{\alpha}A_{56}(\alpha-\xi)d\xi-\frac{q_xR^4}{EI}\int_{\alpha_{x_K}}^{\alpha}A_{56}(\alpha-\xi)d\xi.$$

Substituting value $A_{56}(\alpha-\xi)$ [19] and perfomiing appropriate differentiation and integration taking into account all properties of impulse functions and splines [22, 23], we finally have:

$$B_{11}=-\frac{F_yR^3}{EI}\left\{\frac{\Phi_1(\alpha-\alpha_{F_y})}{a}+\frac{t_1+\gamma^2-3\delta^2}{2a\delta(\gamma^2+\delta^2)}[\gamma\Phi_6(\alpha-\alpha_{F_y})+\delta\Phi_4(\alpha-\alpha_{F_y})]-\right.$$
$$\left.-\frac{t_1+3\gamma^2-\delta^2}{2a\gamma(\gamma^2+\delta^2)}[\gamma\Phi_4(\alpha-\alpha_{F_y})-\delta\Phi_6(\alpha-\alpha_{F_y})]\right\}+\frac{MR^2}{EI}\left\{-\frac{\sqrt{t_1}\Phi_2(\alpha-\alpha_M)}{a}+\right.$$
$$+\frac{t_1+\gamma^2-3\delta^2}{2a\delta(\gamma^2+\delta^2)}[(\gamma^2-\delta^2)\Phi_3(\alpha-\alpha_M)+2\gamma\delta\Phi_5(\alpha-\alpha_M)]-\frac{t_1+3\gamma^2-\delta^2}{2a\gamma(\gamma^2+\delta^2)}[(\gamma^2-$$
$$\left.-\delta^2)\Phi_5(\alpha-\alpha_M)-2\gamma\delta\Phi_3(\alpha-\alpha_M)]\right\}-\frac{q_yR^4}{EI}\left[\frac{\Phi_2(\alpha-\alpha_{y_H})}{a\sqrt{t_1}}+\frac{t_1+\gamma^2-3\delta^2}{2a\delta(\gamma^2+\delta^2)}\Phi_3(\alpha-\right.$$
$$\left.-\alpha_{y_H})-\frac{t_1+3\gamma^2-\delta^2}{2a\gamma(\gamma^2+\delta^2)}\Phi_5(\alpha-\alpha_{y_H})\right]+\frac{q_yR^4}{EI}\left[\frac{\Phi_2(\alpha-\alpha_{y_K})}{a\sqrt{t_1}}+\frac{t_1+\gamma^2-3\delta^2}{2a\delta(\gamma^2+\delta^2)}\Phi_3(\alpha-\right.$$
$$\left.-\alpha_{y_K})-\frac{t_1+3\gamma^2-\delta^2}{2a\gamma(\gamma^2+\delta^2)}\Phi_5(\alpha-\alpha_{y_K})\right]+\frac{F_xR^3}{EI}\left[\frac{\Phi_2(\alpha-\alpha_{F_X})}{a\sqrt{t_1}}+\frac{t_1+\gamma^2-3\delta^2}{2a\delta(\gamma^2+\delta^2)}\Phi_3(\alpha-\right.$$
$$\left.-\alpha_{F_X})-\frac{t_1+3\gamma^2-\delta^2}{2a\gamma(\gamma^2+\delta^2)}\Phi_5(\alpha-\alpha_{F_X})\right]+\frac{q_xR^4}{EI}\left\{-\frac{\Phi_1(\alpha-\alpha_{x_H})}{a}+\frac{t_1+\gamma^2-3\delta^2}{2a\delta(\gamma^2+\delta^2)}[\gamma\Phi_6(\alpha-\right.$$
$$\left.-\alpha_{x_H})+\delta\Phi_4(\alpha-\alpha_{x_H})]-\frac{t_1+3\gamma^2-\delta^2}{2a\gamma(\gamma^2+\delta^2)}[\gamma\Phi_4(\alpha-\alpha_{x_H})-\delta\Phi_6(\alpha-\alpha_{x_H})]\right\}-$$
$$-\frac{q_xR^4}{EI}\left\{-\frac{\Phi_1(\alpha-\alpha_{x_K})}{a}+\frac{t_1+\gamma^2-3\delta^2}{2a\delta(\gamma^2+\delta^2)}[\gamma\Phi_6(\alpha-\alpha_{x_K})+\delta\Phi_4(\alpha-\alpha_{x_K})]-\right.$$
$$\left.-\frac{t_1+3\gamma^2-\delta^2}{2a\gamma(\gamma^2+\delta^2)}[\gamma\Phi_4(\alpha-\alpha_{x_K})-\delta\Phi_6(\alpha-\alpha_{x_K})]\right\}.$$

In a similar way, we calculate the remaining components of the load vector (21), which we do not give here due to the limited volume of the paper.

5 DISCUSSION OF RESEARCH RESULTS

The proposed approach, the essence of which is that before approximation, analytical integration of the differential equation (or equations) is performed, i.e. in one form or another, the transition to integral equations is an alternative to existing numerical methods for solving differential equations. At the same time, not the entire region is subject to discretization, but only its boundary. The inner part of the region is considered as one "element". This leads to a significant decrease in the number of discrete elements, and, consequently, to a decrease in the order of the resolving system of algebraic equations. The method is distinguished by a unified approach to the problems of statics, dynamics and structural stability. The divergence of these three classes of problems consists only in different systems of fundamental functions. The matrix of fundamental functions is very sparse, which significantly improves the stability of numerical operations and ensures high accuracy of the results of the method.

6 CONCLUSIONS

An ordinary sixth-order differential equation is derived with respect to the longitudinal displacement u, which describes the forced oscillations of a circular arch in its plane; the Green function is constructed, the load vector is formed. Thus, the first, analytical component of the numerical-analytical method of boundary elements has been completed. The numerical implementation of the algorithm and the comparison of the results with the results of finite element analysis determine the direction of our further research.

It should be noted that the cost of computer resources when implementing a program for calculating an arch system using the boundary element method is minimal, since it is necessary to solve a system of only twelve algebraic equations, which is significantly less than when using the finite element method.

The results obtained allow us to perform dynamic calculations for forced vibrations of not only arched building structures, but also elements of shipbuilding, engineering and any other structures, the design diagram of which is arches and arched systems of arbitrary configuration.

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