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ON THE BUCKLING OF A ROD UNDER TENSILE FORCE

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Abstract: This article examines the mathematical model of the loss of equilibrium stability of an elastic, rigidly clamped, rectilinear cantilever rod loaded with a longitudinal tensile force applied to a traverse rigidly fixed to the free end of the rod perpendicular to its axis. The study is carried out within the framework of linear rod theory under the standard assumption of small displacements. The way of applying the tensile force allows for frictionless movement of its application point along the traverse, as well as a change in its direction depending on the displacement of the free end of the rod and the slope of the traverse. Unlike the relatively few previously known formulations of problems on the buckling of rectilinear rods caused by tensile forces, this article imposes no restrictions on the distribution of the bending rigidity of the rod along its length. The study is theoretical in nature and is based on a qualitative analysis of the behavior of solutions to a linear differential equation for rod bending. To simplify the analysis, a transition was made from the traditional description of the deformed state in rod theory using a deflection function to a bending moment function. This allowed the solving to the rod buckling problem to be reduced to an eigenvalue problem for a second-order linear homogeneous differential equation for bending moments under homogeneous boundary conditions of a special type. Critical forces are determined as eigenvalues for which a solution to the differential equation exists that is not identically zero and satisfies the boundary conditions. For the constructed boundary problem, a theorem was formulated and proved, which states that with an arbitrary distribution of the bending stiffness of the rod along its length and with the ways of applying the tensile force adopted in the study, a single critical tensile force always exists at which the rod loses stability. The dependence of the critical force on the parameter characterizing the change in the position of the tensile force application point during rod buckling is qualitatively described. An increase in the critical force is also established when the rod is shortened by removing a portion near the clamp and transferring it to the end of the newly formed rod. A comparison of the obtained results with studies, including experimental ones, by other authors showed that the conditions considered in the article can be realized for rods operating in complex structures.

Keywords: tensile rod, loss of stability, boundary problem, critical force, qualitative analysis, existence and uniqueness.

ЩОДО ВТРАТИ СТІЙКОСТІ РОЗТЯГНУТОГО СТРИЖНЯ

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Анотація: У статті розглядається математична модель втрати стійкості рівноваги пружного жорстко зацмленого прямолінійного консольного стрижня, завантаженого поздовжньою розтягуючою силою, прикладеною до поперечної траверси, жорстко закріпленої на вільному кінці стрижня перпендикулярно до його осі. Дослідження здійснюється в межах лінійної теорії стрижней в стандартному припущенні малості переміщень. Спосіб прикладання розтягуючої сили допускає вільне від тертя переміщення уздовж траверси її точки прикладання, а також зміну її напрямку в залежності від переміщення вільного кінця стрижня і нахилу траверси. На відміну від порівняно небагатьох відомих раніше постановок задач про втрату стійкості прямолінійних стрижнів, спричинену розтягуючими силами, у запропонованій статті не накладається обмежень на розподіл згинальної жорсткості стрижня уздовж його довжини. Дослідження має теоретичний характер і спирається на якісний аналіз поведінки розв'язків лінійного диференціального рівняння вигину стрижня. З метою спрощення аналізу був виконаний перехід від традиційного для теорії стрижнів опису деформованого стану з використанням функції прогинів до функції згинальних моментів. Це дозволило привести



розв'язання питання втрати стійкості стрижня до розгляду задачі на власні значення для лінійного однорідного диференціального рівняння другого порядку щодо згинальних моментів за однорідних граничних умов спеціального виду. Критичні сили визначаються як власні значення задачі, для яких існує розв'язок диференціального рівняння, що не дорівнює тотожному нулю і задовольняє граничним умовам. Для побудованої граничної задачі сформульовано і доведено теорему, яка встановлює, що при довільному розподілі згинальної жорсткості стрижня по його довжині при прийнятих у роботі способах прикладання розтягуючої сили завжди існує і до того ж єдина критична розтягуюча сила, при якій стрижень втрачає стійкість. Якісно описана залежність величини критичної сили від параметра, що характеризує зміну положення точки прикладання розтягуючої сили при згинанні стрижня. Встановлено також зростання критичної сили при укороченні стрижня шляхом видалення його частини поблизу защемлення з його перенесенням у кінець новоствореного стрижня. Зіставлення отриманих результатів із дослідженнями, в тому числі експериментальними, інших авторів, показали, що розглянуті у статті умови можуть бути реалізовані для стрижнів як елементів складних конструкцій.

Ключові слова: розтягнутий стрижень, втрата стійкості, гранична задача, критична сила, якісний аналіз, існування і єдиність.

1 INTRODUCTION

Problems of stability, particularly the stability of equilibrium configurations in engineering structures, have attracted the interest of engineers and scientists for nearly three centuries. Stability is a necessary quality of a structure, ensuring its reliable operation under the conditions envisaged by the design. Along technology advanced, the types of engineering structures became more complex and varied, and the intensity and diversity of the impacts on their elements increased, the range of corresponding problems and their solving methods expanded. In most cases in engineering practice, loss of structural stability occurs due to buckling of its elements under compression when the compressive load reaches a certain value, called the critical load. Therefore, the main content of the theory of equilibrium stability for mechanical systems developed to date consists of methods for determining critical loads, as well as methods for controlling them to prevent buckling due to compression. The possibility of buckling under tension is considered questionable by many engineers. At the same time, over the course of several decades, a number of publications have appeared examining the possibility of buckling of tensile members. However, their discussion was illustrative in nature and limited primarily to rods of constant cross-section. The present paper examines the buckling of a rod with an arbitrary distribution of flexural stiffness along its length, caused by a tensile force.

The mathematical content of the problem consists in the construction and qualitative study of a mathematical model of a tensile rod, which is based on a linear differential equation that determines the configuration of the deformed rod. The difference from previous works is the reduction of the problem to an eigenvalue problem for a second-order differential equation with variable coefficients and specific boundary conditions.

The article establishes the existence and uniqueness of the critical force as an eigenvalue of the problem for an arbitrary distribution of bending stiffness and its relationship to certain changes in parameters that characterize the conditions of fixing and application of the tensile load.

2 LITERATURE ANALYSIS AND PROBLEM STATEMENT

The development of formulations and methods for solving problems related to the study of the stability of mechanical systems has a rich and instructive history, reflected in well-known works [1-3]. To date, stability theory has grown into a vast and diversified field of science, rapidly developing alongside the creation of new engineering structures, materials, mathematical methods, and information technologies. The results of this development are reflected in classical works [4-6], as well as in new-generation monographs [7-9], which together represent a virtually exhaustive range of research areas. However, problems of equilibrium stability are considered almost exclusively in relation to the compression of structural elements. In recent decades, the possibility of buckling in thin plates under the action of tensile forces has been established and systematically studied [10-13]. Buckling of a tensile rod is still often regarded as a paradox of no practical significance. Some special cases were described in [6, 14, 15]. One of them, considered in [14] and close to that is studied in the proposed article, is shown in Fig. 1 *a*.

They represent rather elementary models for demonstrating the principal possibility of buckling caused by tensile forces than fragments of any general theory. Only in [15] an attempt was made to give a unified description of a certain set of ways for applying tensile load. However, the possibility of their structural implementation is not systematically discussed. Refusing to pursue any practical goals allowed to limit the considerations to rods with constant flexural rigidity along their length. Article [16] describes an interesting case of

buckling of a tensile rod formed from two parts connected in a special manner, allowing free relative transverse displacements of the connected sections (Fig. 1 *b*).

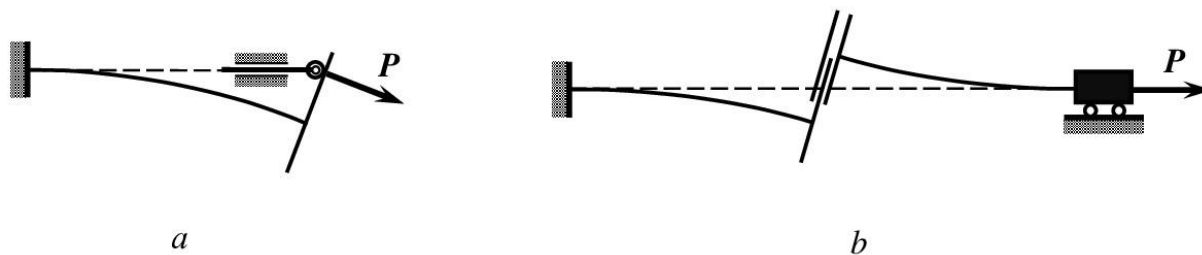


Fig. 1. Examples of loss of stability under tensile forces: *a* – by [14], *b* – by [16]

The bending of each individual part can be described as rod buckling, which is studied in this article. The buckling effect described in [16] is clearly demonstrated using a physically constructed model. A corresponding video is freely available online.

The present study uses a standard criterion of static stability ([15]), based on the existence of a non-trivial solution to the differential equation for bending of a tensile rod and on a qualitative study of the properties of solutions of this equation under the corresponding boundary conditions, since an arbitrary distribution of bending stiffness does not allow this solution to be expressed in an exact form.

3 THE PURPOSE AND OBJECTIVES OF THE STUDY

The objective of this work is to study the buckling conditions of a rectilinear cantilever rod loaded by a longitudinal tensile force constant along its length (Fig. 2).

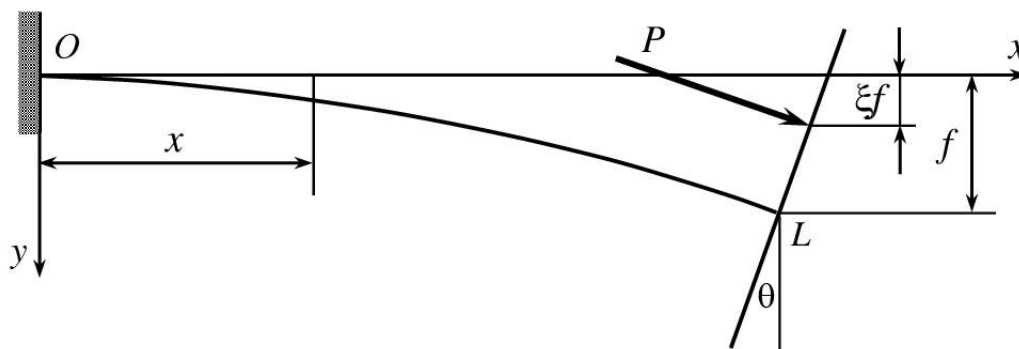


Fig. 2. Schematic buckling form of the rod under study

In extension of the previously considered formulations [6, 14, 15, 16], an arbitrary change in the bending stiffness along the length of the rod is allowed. In addition, it is assumed that the way of applying the tensile load at the free end allows a certain dependence of its direction and application point on the deflection and angle of rotation of the end cross-section. This way is one of those considered in [15] and, in principle, can be implemented without violating conservatism. As it turns out, this is precisely the way implemented for each of the two rods in Fig. 1 *b*. The proposed generalizations are motivated by the desire to study the conditions which may occur for tensile rod as an element of a compound structure.

4 RESEARCH RESULTS

4.1. Mathematical model description. We consider a cantilever rod OL , rigidly fixed at one end and having a traverse at its free end – an absolutely rigid transverse rod perpendicular to the axis of the bending rod (Fig. 2). It is assumed that the traverse is perfectly smooth, which implies that equilibrium of the rod loaded with force $\mathbf{P}$ is possible only when force $\mathbf{P}$ and the traverse are perpendicular.

The following notations are used:

$B = B(x) = EJ$ – bending stiffness of a rod, variable along its length;

ℓ – rod length;

P – the magnitude of the longitudinal tensile force perpendicular to the plane of the end section L of the rod;

$y = y(x)$ – deflection of a rod in section at a distance x from the clamped end;

$\varphi = \varphi(x)$ – slope of the section at a distance x from the clamped end;

f, θ – respectively, the deflection and the slope of the end section L of the rod;

$M = M(x)$ – bending moment in the section x .

ξ – the eccentricity coefficient of the tensile force, which determines the transverse displacement of its point of application at bending of the rod; it is accepted that $-\infty < \xi < 1$; at $\xi = 0$ we will call the rod centrally loaded.

The consideration is based on the linear theory of rods, which assumes small displacements and deformations.

The differential equation that determines the configuration $y(x)$ of a deformed rod in a linear approximation, taking into account the slope of the force $\mathbf{P}$, has the form

$$By'' = -M(x) = P\theta(\ell - x) - P(\xi f - y). \quad (1)$$

Here and below, the prime denotes the derivative with respect to the coordinate.

Differentiating both sides with respect to x , we obtain an equation for the slope $\varphi = \varphi(x) = y'$

$$(B\varphi)' = P\varphi - P\theta. \quad (2)$$

By introducing a new variable (bending moment) $M = -B\varphi'$ and again differentiating with respect to x , we get at a homogeneous equation

$$M'' = (P/B)M. \quad (3)$$

As can be seen from Fig. 2, the deflections and angles of inclination of the ends of the rod satisfy the relations

$$\varphi(0) = 0, \quad \varphi(\ell) = \theta, \quad y(0) = 0, \quad y(\ell) = f. \quad (4)$$

From (1) and (2) it follows

$$M(0) = -P\theta\ell + P\xi f, \quad M(\ell) = -Pf(1 - \xi), \quad M'(0) = P\theta, \quad M'(\ell) = 0, \quad (5)$$

from where, excluding f and θ , we obtain the homogeneous boundary conditions

$$M'(\ell) = 0, \quad M(0) + M'(0)\ell = -\frac{\xi}{1-\xi}M(\ell), \quad (6')$$

complementary to the equation (3).

4.2. Main theorem. The main mathematical result of the work is expressed by the following theorem.

Theorem (on existence and uniqueness of critical force). For any ξ , $-\infty < \xi < 1$, there is a unique value of $P = P(\xi)$, which corresponds to a nonzero solution of the differential equation (3), satisfying the boundary conditions (6). When ξ increases from $-\infty$ to 1 the value $P = P(\xi)$ increases monotonically from 0 to ∞ .

To prove the theorem, we establish a number of preliminary results.

4.3. Auxiliary results. Let us consider the solution $M(x)$ of equation (3) for some fixed P , satisfying only the first of the conditions (6), and accept $M(\ell) = \text{const.} > 0$.

In the interval $0 \leq x \leq \ell$, $M(x)$ retains its sign. Otherwise, there would exist x_1 , $0 < x_1 < \ell$, such that $M(x_1) = 0$ and $M(x) > 0$ at $x_1 < x \leq \ell$, from which it would follow that there exists such x_2 , $x_1 < x_2 < \ell$, for which $M'(x_2) > 0$, from which, in turn, it would follow that there exists such x_3 , $x_2 < x_3 < \ell$, for which $M''(x_3) < 0$, in contradiction with (3). Thus, $M(\ell) > 0 \Rightarrow M(x) > 0$ and on the basis of (3) $M''(x) > 0$ for everyone x , $0 \leq x \leq \ell$. This means that the graph of the function $M(x)$ is concave upwards (see Fig. 3).

From (3) the following relations follow

$$M'(x) = M'(x) - M'(\ell) = P \int_{\ell}^x \frac{M(s)}{B(s)} ds = -P \int_x^{\ell} \frac{M(s)}{B(s)} ds < 0, \quad (7)$$

$$x_2 > x_1 \Rightarrow M'(x_2) - M'(x_1) = \int_{x_1}^{x_2} M''(s) ds = P \int_{x_1}^{x_2} \frac{M(s)}{B(s)} ds > 0,$$

that allow us to conclude that the derivative $M'(x)$ on the interval of interest to us is negative and increases with increasing x , and that $M(x)$ decreases with increasing x .

For an arbitrary function $M(x)$ the identity holds

$$M(x) = M(\ell) - M'(\ell)(\ell - x) + \int_x^{\ell} M''(s)(s - x) ds,$$

which can be verified by calculating the integral by parts.

From here, taking into account (3) and the initial condition (6), we arrive at the equality

$$M(x) = M(\ell) + P \int_x^{\ell} \frac{M(s)}{B(s)} (s - x) ds. \quad (8)$$

From relation (7) we find

$$M'(0) \cdot \ell = -P \int_0^{\ell} \frac{M(s)}{B(s)} \cdot \ell \cdot ds, \quad (9)$$

which in combination with (8) gives

$$M(0) + M'(0)\ell = M(\ell) + P \int_0^{\ell} \frac{M(s)}{B(s)} (s - \ell) ds. \quad (10)$$

The bending moment M as a function of P increases with increasing P . To verify this, let us consider two functions M and M_1 , corresponding respectively to the values P and P_1 , $P_1 > P$, at $M_1(\ell) = M(\ell) > 0$. Due to the continuity of the second derivative $M''(x)$ and to the equation (3), there is a certain neighborhood of the right end L of the rod, where $M_1(x) > M(x)$. If we assume that for some x the opposite inequality $M_1(x) \leq M(x)$ holds, there must exist such x^* that $M_1(x^*) = M(x^*)$ and $M_1(x) > M(x)$ for $x^* < x < \ell$ in contradiction with equality (8). The same equality shows that the growth of M as a function of P is unlimited.

4.4. Proof of the theorem. The right side of (10) decreases monotonically from $M(\ell)$ to $-\infty$ as P increases from 0 to ∞ . Therefore, the expression $M(0) + M'(0)\ell$ in the boundary condition (6) also decreases monotonically from $M(\ell)$ to $-\infty$, becoming exactly once equal to $-(\xi/1-\xi)M(\ell)$ for a given ξ , $-\infty < \xi < 1$. This means that for any ξ of the range $-\infty < \xi < 1$ there is a unique value P for which there is a non-trivial solution to equation (3) that satisfies both boundary conditions (6). This value represents the only critical force of the considered rod loaded with a tensile longitudinal force

The expression $M(0) + M'(0)\ell$ graphically represents the ordinate of the point of intersection of the tangent to the graph of the function $M(x)$, drawn at $x=0$, with the vertical $x=\ell$ (Fig. 3). As P increases, this point moves downwards, monotonically and unlimitedly. The expression $-(\xi/1-\xi)M(\ell)$ decreases monotonically from $M(\ell)$ to $-\infty$ as ξ increases from $-\infty$ to 1. A larger value ξ corresponds to a larger value P satisfying the second condition (6), i.e. the critical force of the rod increases monotonically with the growth of the coefficient ξ . The theorem is proven.

4.5. Corollary. Shortening a rod clamped at one end, with a corresponding transfer of the fixation conditions to the end of the shortened rod, leads to an increase in the critical force, regardless of the law of distribution of bending rigidity along the length of the rod.

Proof. Any value $P > 0$ is the critical force of the rod under consideration at certain value $\xi = \xi(P)$ between $-\infty$ and 1. This value is uniquely determined by the point of intersection of the tangent to the curve $M(x)$ at the point $x=0$ with the vertical $x=\ell$ (Fig. 3).

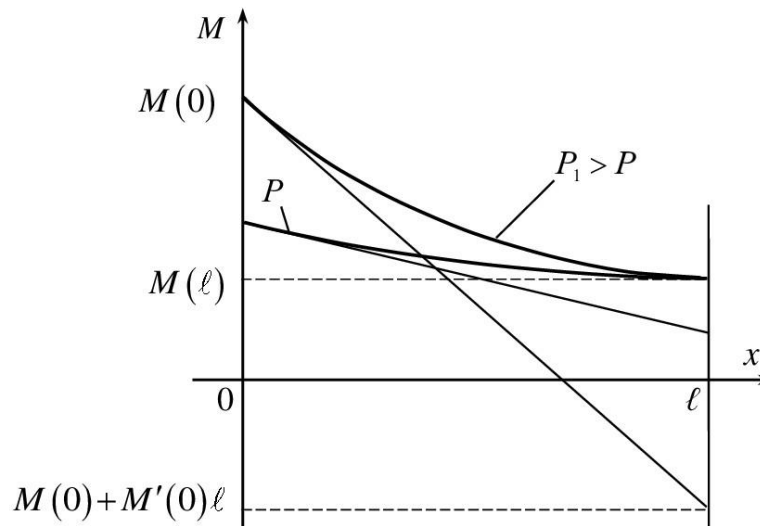


Fig. 3. Schematic graphs of the function $M(x)$, $P_1 > P$

If we draw another tangent to the same curve at the point $x' > 0$, then due to the absence of inflection points on the curve, the corresponding point of intersection with the vertical $x = l$ will be higher, and it will correspond to the value $\xi' < \xi(P)$. This means that the critical force P of a rod clamped at the left end $x = 0$ is at the same time the critical force of a shortened rod clamped in section x' , but with a smaller coefficient ξ . Since, as established above, growth ξ leads to an increase in the critical force, it can be concluded that when $\xi = \xi(P)$ the shortened rod will have a critical force $P' > P$. Thus, the well-known result for the critical forces of compressed rods is established for a rod in tension. The corollary is proven.

4.6. On the physical realization of the loss of stability of a rod under tension This paper does not consider questions of structural implementation of ways for applying tensile forces in which the coefficient ξ will be nonzero. Such conditions may arise in structures containing, among other, tensile rods.

In [16], the behavior of a rod composed of two elastic rods similar to the one considered above was investigated. Each of these rods is rigidly clamped at one end and has a traverse at the opposite free end – an absolutely rigid rod element perpendicular to the axis of the rod. The rods are connected at their free ends using a slider – a device that equalizes the slopes of the connected end cross sections, but allows their relative sliding without friction in a direction perpendicular to the axes of the deformed rods at their connection (Fig. 1b, Fig. 4).

At buckling, the rods act on each other with systems of forces distributed over the contact surface of the perfectly smooth traverses and directed perpendicular to the traverses. Both systems have resultants applied at some point K on this surface, which, according to the well-known law of equality of action and reaction, are equal in magnitude and opposite in direction. The point K must be located between the end sections L_1 and L_2 of the rods. Otherwise, both rods would have the same curvature sign.

This allows us to consider each of the rods as a rod shown in Fig. 2, with the corresponding value of ξ . If the point K does not lie on the axis of the undeformed rod (the dotted line in Fig. 4), it will be displaced from this axis in the same direction as the end of one of the rods. If this is L_1 , then, as can be seen from the figure, $\xi_1 > 0$, $\xi_2 < 0$.

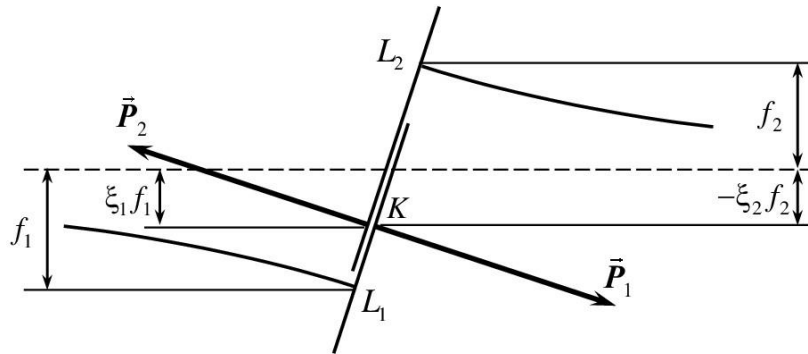


Fig. 4. Interaction of rods in a structure shown in Fig. 1 b

Let us denote by P_0 the critical force of a centrally loaded rod (for which $\xi = 0$). When $\xi < 0$ the inequality $P < P_0$ is satisfied, and when $\xi > 0$ the inequality $P > P_0$ is satisfied.

It was established above that the critical force increases monotonically with increasing ξ . This means that the critical force P of the composite rod satisfies the relations

$P_{10} < P < P_{20}$, where P_{10} and P_{20} are, respectively, the critical forces of the left and right rods at $\xi_1 = \xi_2 = 0$.

4.6. Expansion of the fixing conditions. In the previous discussion, it was assumed that the clamping of the left end O of the rod is absolutely rigid. It can be verified that all the stated conclusions remain valid even for finite clamping rigidity. In this case, in (4), instead of the boundary condition $\varphi(0) = 0$, the condition $M(0) = -\alpha\varphi(0)$, $\alpha = \text{const.} > 0$, should be adopted. In (5) instead of $M'(0) = P\theta$ you should write $M'(0) = P\theta - P\varphi(0)$. After excluding f and θ the second of the conditions (6) will take the form

$$M(0) + M'(0)\ell - \frac{P}{\alpha}M(0) = -\frac{\xi}{1-\xi}M(\ell). \quad (6)$$

As we see, the left-hand side of (6'), as in (6), decreases monotonically with increasing P , which implies the existence and uniqueness of the critical force, which increases monotonically with increasing ξ . We also note that the left-hand side of (6') increases with increasing α , which implies that, all other things being equal, the critical force increases monotonically with increasing clamping rigidity α .

5 RESEARCH RESULTS DISCUSSION

The results presented in this article confirm and, to some extent, complement those established in previously published studies on the possibility of buckling of rods caused by tensile forces.

The fact of loss of stability of rod under tension for an arbitrary distribution of bending rigidity along its length has been established.

The existence and uniqueness of a critical tensile force for a rod under certain conditions of fixation and loading is established.

The dependence of the critical force on the conditions of fixation and longitudinal loading of the rod was investigated. Specifically, the monotonicity of the critical force as a

function of the parameter ξ characterizing the position of the longitudinal force application point was established.

A connection is noted between the problems of buckling of an individual tensile rod and a compound structure, previously studied independently.

6 CONCLUSIONS

The studies presented in this paper complement and expand on the still few and unsystematized descriptions of buckling of rods under the action of tensile forces.

The obtained results can be applied in studying the behavior of mechanical systems containing longitudinally loaded rods of variable stiffness, as well as for control the effectiveness of numerical algorithms.

It can be expected that they will be useful in the transition to studying the stability of a broader class of rod systems whose elements operate under longitudinal loading and can be applied in the future when solving more complex problems of control and optimization of the mechanical characteristics of various engineering structures.

7 ETHICAL DECLARATIONS

The proposed publication does not pursue any material interests and is intended only to expand and deepen knowledge concerning some topical problems of mechanics, and their sharing among specialists in relevant scientific fields.

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